Reminder VIII

- M oriented = M orientable and orientation fixed.

$(U_\alpha, \varphi_\alpha)_\alpha$ a maximal atlas preserving the orientation.

- $\phi \in \Lambda^m(M)$ with supp $\phi \subset U_\beta$. Then

$\phi_p = a(p)\,(dx^1)_p \wedge \dots \wedge (dx^m)_p \quad \forall p \in U_\beta$

and for $a \in C^\infty(U_\beta)$, and

integral of a m-form

$$\underline{\int_M \phi} = \int_{U_\beta} \phi := \int (a \circ \varphi_\beta^{-1})(x^1, \dots, x^m)\, dx^1 \dots dx^m.$$

independent of the chart

For general $\phi \in \Lambda^m(M)$, $\int \phi := \sum_\beta \int_{U_\beta} \underbrace{f_\beta \phi}_{\in \Lambda^m(U_\beta)}$

with $\{f_\beta\}_\beta$ a partition of unity subordinated to the covering.

- Stokes' theorem : $\int_{\partial M} \iota^*(\phi) = \int_M d\phi$, $\phi \in \Lambda^{m-1}(M)$.

- For $f \in \Lambda^0(M) \equiv C^\infty(M)$, $\int_M f = \int_M f \phi$ fixed m-form

- For $c : [a,b] \to M$ with $C = c([a,b])$, and
curve

local representation

$\phi \in \Lambda^1(M)$, $\int_C \phi = \int_a^b c^*\phi \equiv \int_a^b a(t)\, dt$.

integral along a curve

- If $\phi \in \Lambda^0(M)$, then $\int_C d\phi = \phi(c(b)) - \phi(c(a))$.

- If $C_1 \sim C_2$ (Continuously deformed) and $\phi \in \Lambda^1(M)$ with $d\phi = 0$, then $\int_{C_1} \phi = \int_{C_2} \phi$.

Riemannian metric

- <u>Riemannian manifold</u> : $\phi \in \Sigma^2(M)$, positive definite ("positive definite matrix at any $p \in M$")
- Any smooth manifold admits a Riemannian metric