Reminder VII

- For M of dim m, $r \in \mathbb{N}$, $\Lambda^r(M) \ni \phi$ an <u>exterior differential form</u>, $\phi : M \ni p \mapsto \phi_p \in \mathfrak{I}^r(T_p(M))$.

$\Lambda(M) := \bigoplus_{r=0}^{m} \Lambda^r(M)$ <u>exterior algebra</u>, with pointwise addition and wedge product.

- For (U, φ) a chart, $\{ (dx^{i_1})_p \wedge \ldots \wedge (dx^{i_r})_p \mid 1 \leq i_1 < \ldots < i_r \leq m \}$ a basis of $\Lambda^r(T_p(M))$.

- $\exists!\ d : \Lambda(M) \to \Lambda(M)$ s.t. 1) $\underset{\Lambda^1(M)}{df} = \overset{\text{differential of } f}{df}$ $\forall f \in \underset{C^\infty(M)}{\Lambda^0(M)}$

2) $d(\overset{\Lambda^r(M) \ni}{\phi} \wedge \overset{\in \Lambda^s(M)}{\psi}) = (d\phi) \wedge \psi + (-1)^r \phi \wedge (d\psi) \in \Lambda^{r+s}(M)$

3) $d^2 = 0$. $\quad d =$ <u>exterior derivative</u>

d local + explicit formula.

- <u>Orientable manifold</u> if $\exists$ maximal atlas s.t. all transition maps $\varphi_\alpha \circ \varphi_\beta^{-1}$ have a positive Jacobian det.

$\Leftrightarrow \exists\ \phi \in \Lambda^m(M)$ which never vanishes.

- Smooth manifold with boundary ∂M $\leftarrow$ smooth manifold of dim $m-1$

- If M orientable, then ∂M orientable (inward or outward orientation).
- Locally finite cover and smooth partition of unity: $\{f_\alpha\}_\alpha \subset C^\infty(M)$, $f_\alpha \geq 0$, $\{\mathrm{supp}\, f_\alpha\}$ locally finite, $\sum_\alpha f_\alpha = 1$.