Reminder VI

- <u>Tensor algebra</u> $\mathcal{T}(V) := \mathcal{T}^0(V) \oplus \mathcal{T}^1(V) \oplus \mathcal{T}^2(V) \oplus \dots$

of dimension ∞

of dim n $\quad \mathcal{T}^0(V) = \mathbb{R}$

with componentwise addition and product $\otimes$

<u>Exterior algebra</u> $\Lambda(V) := \bigoplus_{j=0}^{n} \Lambda^n(V)$ with componentwise

$\Lambda^0(V) = \mathbb{R}$

addition and wedge product $\wedge$, of dimension 2^n.

- <u>Dual basis</u> = basis on V^* satisfying $\xi_j(E_i) = \delta_{ij}$

basis of V^* $\quad$ basis of V

For the coordinate frame $\{E_{j,p}\}_{j=1}^{m}$ (basis of $T_p(M)$), the dual basis is denoted $\{(dx^i)_p\}_{i=1}^{m}$.

coordinate coframe

- For $f \in C^\infty(p)$, $(df)_p(X_p) := X_p(f)$, $(df)_p \in T_p^*(M)$

and $(df)_p = \sum_{j=1}^{m} \frac{\partial(f \circ \varphi^{-1})}{\partial x^i}(\varphi(p))\,(dx^i)_p$,

For $f \in C^\infty(M)$, df is called the <u>differential of f</u>.

- (r,s)-tensor field : $\phi : M \ni p \mapsto \phi_p \in \mathcal{T}^r_s(T_p(M))$

$\mathcal{T}^r_s(M)$ smooth (r,s)-tensor field.

vector field

$\mathcal{T}^0_1(M) \equiv \mathfrak{X}(M)$, $\mathcal{T}^1_0(M) = \{\text{covector field}\} \ni df$

$\mathcal{T}^2_0(M) = \{\text{field of bilinear map } T_p(M)\}$

- $\Sigma^r(M) = \{$ smooth symmetric $(r,0)$-tensor field $\}$

  $\Lambda^r(M) = \{$ smooth antisymmetric $(r,0)$-tensor field $\}$

- If $\phi \in \Sigma^2(M)$, positive definite, then

  M is a <u>Riemannian manifold</u>, ϕ the Riemannian metric.