Reminder IV

- With (U, φ) a chart, define $\left\{ \varphi_* \left(\frac{\partial}{\partial x^j} \Big|_{\varphi(p)} \right) \right\}_{j=1}^{m}$ a basis of $T_p(M)$, called <u>coordinate frame</u>.
- Each curve defines a tangent vector (and vice-versa):

coordinate frame

$$c_* \left(\frac{d}{dt}\Big|_t \right) = \sum_{j=1}^{m} (c^j)'(t) \; E_{j, c(t)}$$

local coordinates of the curve

- <u>Tangent bundle</u> $T(M) = \bigcup_{p \in M} T_p(M)$, and <u>vector field</u> $X : M \to T(M)$, $p \mapsto X_p \in T_p(M)$

regularity defined locally, using coordinate frames.

Smooth vector fields : $\underline{\mathfrak{X}(M)}$.

$\mathfrak{X}(M)$: vector space, $C^\infty(M)$-module, Lie algebra

- Given $X \ni \mathfrak{X}(M)$, $p \in M$, $\exists$ <u>integral curve</u> c_p with $\dot{c}_p(t) \equiv (c_p)_* \left(\frac{d}{dt}\Big|_t \right) = X_{c(t)}$, $c(0) = p$.

maximal domain for c_p

- For more than 1 p : $W := \bigcup_{p \in M} I_p \times \{p\}$

Flow $\Theta : W \to M$ s. t. $\Theta(0, p) = p$ and $\Theta(t, \Theta(s, p)) = \Theta(s+t, p)$

also called 1 parameter group action on M.