

6 pts

Exercise 1 Compute the derivative of the function $(0, \infty) \ni x \mapsto x^{x^2} \in \mathbb{R}$:

2

$$\begin{aligned} (x^{x^2})' &= (e^{x^2 \ln(x)})' = e^{x^2 \ln(x)} \left(2x \ln(x) + x^2 \frac{1}{x} \right) \\ &= \underline{e^{x^2 \ln(x)} (2x \ln(x) + x)} \end{aligned}$$

Exercise 2 Compute the following limits:

3

$$1. \lim_{x \rightarrow 0} \frac{\sin(x) \sin(2x) \sin(3x)}{x^3} = \lim_{x \rightarrow 0} \frac{\sin(x)}{x} \cdot 2 \cdot \frac{\sin(2x)}{2x} \cdot 3 \cdot \frac{\sin(3x)}{3x} = \underline{6}$$

$$2. \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \lim_{x \rightarrow 0} \frac{e^x}{1} = \underline{1}$$

↑ (H)

$$3. \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{2x} = \lim_{x \rightarrow 0} \frac{e^{2x}}{2} = \underline{\underline{\frac{1}{2}}}$$

↑ (H) ↑ (H)

Exercise 3 Define $\tanh(x) := \frac{\sinh(x)}{\cosh(x)}$ for any $x \in \mathbb{R}$. Compute the derivative of the function \tanh .

1

$$\begin{aligned} \tanh(x)' &= \left(\frac{\sinh(x)}{\cosh(x)} \right)' = \frac{\cosh(x)^2 - \sinh(x)^2}{\cosh(x)^2} \\ &= \underline{\underline{1 - \tanh(x)^2}} \quad \text{or} \quad \underline{\underline{\frac{1}{\cosh(x)^2}}} \end{aligned}$$