

Total : 5 pts

Nagoya University, G30 program

Fall 2024

Math tutorial Ia

Instructor : Serge Richard

Quiz I

Name : _____

Exercise 1 Compute

1 a) $\lim_{x \nearrow 2} \frac{(2-x)}{|2-x|} = \lim_{x \nearrow 2} \frac{2-x}{2-x} = \underline{1}$

1 b) $\lim_{n \rightarrow \infty} \sqrt{n^2 + n - 2} - n = \lim_{n \rightarrow \infty} \frac{n^2 + n - 2 - n^2}{\sqrt{n^2 + n - 2} + n}$
 $= \lim_{n \rightarrow \infty} \frac{n - 2}{n \sqrt{1 + 1/n - 2/n^2} + n} = \underline{\underline{\frac{1}{2}}}$

$1 \frac{1}{2}$ Exercise 2 Consider a sequence $(b_n)_{n \in \mathbb{N}}$. Give the definition of the convergence of this sequence when $n \rightarrow \infty$.

$\exists b_\infty \in \mathbb{R}$ such that for any $\varepsilon > 0$, there exists $N \in \mathbb{N}$ with

$$|b_n - b_\infty| \leq \varepsilon \quad \forall n \geq N.$$

$1 \frac{1}{2}$ Exercise 3 Which of the following functions $f : \mathbb{R} \rightarrow \mathbb{R}$ is even or odd:

a) $f(x) = x^5 + 2x$,

odd

b) $f(x) = \cos(x) - \sin(x)$,

not even, not odd

c) $f(x) = |x|^3 - x^2 + 1$

even

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Exercise 1 Compute

1 a) $\lim_{x \nearrow 1} \frac{(x-1)}{|x-1|} = \lim_{x \nearrow 1} \frac{x-1}{-(x-1)} = \underline{-1}$

1 b) $\lim_{n \rightarrow \infty} \sqrt{n^2 + 3n - 2} - n = \lim_{n \rightarrow \infty} \frac{(n^2 + 3n - 2) - n^2}{\sqrt{n^2 + 3n - 2} + n}$
 $= \lim_{n \rightarrow \infty} \frac{3n - 2}{n\sqrt{1 + 3/n - 2/n^2} + n} = \underline{\frac{3}{2}}$

1 $\frac{1}{2}$ Exercise 2 Consider a sequence $(a_n)_{n \in \mathbb{N}}$. Give the definition of the convergence of this sequence when $n \rightarrow \infty$.

$\exists a_\infty \in \mathbb{R}$ such that for any $\varepsilon > 0$, there exists $N \in \mathbb{N}$ with

$$|a_n - a_\infty| \leq \varepsilon \quad \forall n \geq N.$$

1 $\frac{1}{2}$ Exercise 3 Which of the following functions $f : \mathbb{R} \rightarrow \mathbb{R}$ is even or odd:

a) $f(x) = \cos(x) + \sin(x)$,

b) $f(x) = x^3 + 2x$,

c) $f(x) = |x|^3 - x^2 + 1$

not even, not odd

odd

even