Total : 7 pts

Nagoya University, G30 program Fall 2024

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Quiz I Name : ____________________

Exercise 1 *Complete the following sentence as precisely as possible: The function $f:(0,1)\to\mathbb{R}$ has a limit at 0 if* there exists $f(0)\in\mathbb{R}$ such that for any $\varepsilon>0$

2 pts

we can find $\delta>0$ with $|f(x)-f(0)|\leq\varepsilon$ $\forall\ x\leq\delta$.

Exercise 2 *Consider $f:\mathbb{R}\to\mathbb{R}$ and $x_0\in\mathbb{R}$. Circle the letter(s) which correspond(s) to a correct statement:*

(a) *If $\lim_{h\to0} f(x_0+h)$ exists, then f is continuous at x_0,*

(b) *f is continuous at x_0 if $\lim_{x\searrow x_0} f(x)=\lim_{x\nearrow x_0} f(x)$,*

(c) *If $\lim_{x\to x_0} f(x)=f(x_0)$, then f is continuous at x_0,*

3 pts

(d) *If f is the difference of two continuous functions, then f is continuous,*

(e) *If $f(x)=\big(\sin(x)\big)^2$, then f is continuous,*

(f) *If $f(x)=\sum_{n=0}^{\infty}\frac{1}{n!}x^n$, then f is continuous.*

Exercise 3 *Circle the correct properties:*

(i) $[0,\pi]\ni x\mapsto\cos(x)\in\mathbb{R}$ (a) *surjective* (b) *injective*

(ii) $\mathbb{R}\ni x\mapsto\cos(x)\in[-1,1]$ (a) *surjective* (b) *injective*

2 pts

(iii) $[0,\pi/2]\ni x\mapsto\cos(x)\in[-1,1]$ (a) *surjective* (b) *injective*

(vi) $[0,\pi]\ni x\mapsto\cos(x)\in[-1,1]$ (a) *surjective* (b) *injective*