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Exercise 1.2.2. *Specialize the formula (1.2.1) for any absolutely continuous random variable, as presented in Definition 1.1.10, or for a discrete valued random variable, as presented in Definition 1.1.11, when $\Lambda \subset \mathbb{R}$.*

$$\mathbb{E}(f(X)) := \int_\Lambda f(x)\,\mu_X(\mathrm{d}x). \tag{1.2.1}$$

For any absolutely continuous random variable $X : \Omega \to \mathbb{R}$, there exist a (measurable) function $\pi_X : \mathbb{R} \to [0,\infty)$ satisfying for any $A \in \sigma_B$, related to the induced probability measure $\mu_X : \sigma_B \to [0,1]$ by

$$\mu_X(A) = \int_A \pi_X(x)\,dx$$

as defined in Definition 1.1.10. Thus, the corresponding expectation of a measurable function $f : \mathbb{R} \to \mathbb{R}$ is,

$$\mathbb{E}(f(X)) = \int_{\mathbb{R}} f(x)\,\mu_X(dx)$$
$$= \int_{\mathbb{R}} f(x)\,\pi_X(x)\,dx$$

For any discrete valued random variable $X : \Omega \to \Lambda$, $X(\Omega) = \{X(\omega) \mid \omega \in \Omega\} \subset \mathbb{R}$ is finite or countable, and has the probability mass function $p_X : X(\Omega) \to [0,1]$ by $p_X(x) = \mathbb{P}(X^{-1}(\{x\}))$ for any $x \in X(\Omega)$, which satisfies $\sum_{x \in X(\Omega)} p_X(x) = 1$, as defined in Definition 1.1.11. Thus, the corresponding expectation of a measurable function $f : \Lambda \to \mathbb{R}$ is,

$$\mathbb{E}(f(X)) = \int_\Lambda f(x)\,\mu_X(dx)$$
$$= \sum_{x \in X(\Omega)} f(x)\,p_X(x)$$