Reminder IX

- Wiener integral : if $f \in L^2([0,T])$, then

$Y_t := \int_0^t f(u)\, dB_u$ defines a Gaussian process $Y = (Y_t)_t$.

- For $X \in M^2_{loc}([0,T])$, $Y_t := \int_0^t X_u\, dB_u$,

$Y = (Y_t)_{t \in [0,T]}$ has finite quadratic variation.
and is continuous + infinite variation in general.

- Itô's lemma : $f \in C^2(\mathbb{R}^2)$, then

- $$f(t, B_t) = f(0,0) + \int_0^t [\partial_x f](s, B_s)\, dB_s + \int_0^t \{[\partial_t f](s, B_s) + \tfrac{1}{2}[\partial_x^2 f](s, B_s)\}\, ds$$

- N-dimensional analog : only in terms of B... too restrictive!

$$f(t, B_t) = f(0,0) + \int_0^t [\nabla f](s, B_s)^T dB_s + \int_0^t \{[\partial_t f](s, B_s) + \tfrac{1}{2}[\Delta f](s, B_s)\}\, ds$$

- Itô processes : $V \in M^2_{loc}([0,T])$, $D \in M^1_{loc}([0,T])$,

adapted to B. Set $X_t := X_0 + \int_0^t V_s\, dB_s + \int_0^t D_s\, ds$.

Then $X = (X_t)_{t \in [0,T]}$ is adapted to B.

New notation : $df(t, B_t) = [\partial_x f](t, B_t)\, dB_t + \{[\partial_t f](t, B_t) + \tfrac{1}{2}[\partial_x^2 f](t, B_t)\}\, dt$

or $dX_t := V_t\, dB_t + D_t\, dt$. + initial condition