

Reminder VII

- Martingale : $E(M_t - M_s | \mathcal{F}_s) = 0$ and $E(M_t) = ct$
- Stopping time : random variable T satisfying $\{T \leq t\} \in \mathcal{F}_t$.
↑ example : First entry / hitting time, exit time ...
- Stopped martingales $M_{t \wedge T}$ are also martingales.
- For discrete time martingales :
Optional stopping thm : $E(M_T) = E(M_0)$ if T bounded,
Almost sure convergence of positive martingale.
- Dominated convergence thm are useful!
- Few results about stopping time for 1D Brownian motion, and about prob of going through 0 in (t, T) .
- Reminder on Riemann integrals
- Riemann - Stieltjes integrals : $g, F : [a, b] \rightarrow \mathbb{R}$
$$S(g, F, \mathcal{P}_\epsilon, \mathbf{c}) := \sum_{j=1}^{n_\epsilon} g(c_j) (F(t_j^p) - F(t_{j-1}^p))$$
- 2 criteria for convergence $|\mathcal{P}_\epsilon| \rightarrow 0$; p -variation.