Reminder VI

- Jensen inequality ← convex functions

- If $Y : \Omega \to \Lambda'$ is a second r.v. then

σ-algebra generated by Y — Doob-Dynkin lemma

$$\mathbb{E}(X|Y) := \mathbb{E}(X|\sigma(Y)) = g(Y) \quad \text{for } g : \Lambda' \to \Lambda$$

conditional expectation

- If $X \in L^2(\Omega, \mathcal{F}, \mathbb{P})$, then $\mathbb{E}(X|\mathcal{G}) \in L^2(\Omega, \mathcal{G}, \mathbb{P})$

best approximation of X by elements of $L^2(\Omega, \mathcal{G}, \mathbb{P})$

- <u>Conditional probability</u> : $\{\nu_y\}_{y\in\Lambda'}$ measures on $(\Lambda, \mathcal{E})$

satisfying $\mathbb{P}(X \in A, Y \in B) = \int_B \nu_y(A)\, \mu_Y(dy)$

Then $\mathbb{E}(X|Y) = g(Y)$ with $g(y) = \int_\Lambda x\, \nu_y(dx)$

conditional expectation

<u>mean value</u> of conditional probability

$\shortparallel$

expectation

Other notation : $\mathbb{E}(X | Y = y) \equiv \nu_y$.

joint density

- If $\mu_{(X,Y)}$ <u>abs. cont.</u>, $\nu_y(x) = \begin{cases} \dfrac{\pi_{(X,Y)}(x,y)}{\pi_Y(y)}, & y \notin Q \\ \tilde{\pi}(x) & y \in Q \end{cases}$ ← marginal

any density

with $Q = \{y \mid \pi_Y(y) = 0\}$

- <u>Martingale</u> : $(\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{t\in T}, (M_t)_{t\in T})$,

$M_t \in L^1(\Omega, \mathcal{F}, \mathbb{P})$, $\mathbb{E}(M_t | \mathcal{F}_s) = M_s \quad \forall s \leq t$.

Similarly, <u>supermartingale</u> and <u>submartingale</u>.