Reminder V

- <u>Variation</u> of a function on an interval
- For 1D Brownian motion, $t \longmapsto B_t(\omega)$ is <u>continuous</u> a.s., has an <u>infinite variation</u> a.s., is <u>nowhere differentiable</u> a.s. But $\{B_t\}_{t \geq 0}$ has a <u>L^2-convergent quadratic variation</u>.
- ND Brownian motion = N independent 1D Brownian m.
- Conditional prob. and expectation for discrete valued r.v.

$X : \Omega \to \Lambda$ standard mea-space

- <u>Conditional expectation of X given $\mathcal{G}$</u> $\subset \mathcal{F}$ $\stackrel{\text{def}}{=}$ $\mathcal{G}$-measurable r.v. $\mathbb{E}(X \mid Y)$ satisfying $\forall D \in \mathcal{E}$

$$\int_D \mathbb{E}(X \mid \mathcal{G}) \, d\mathbb{P} = \int_D X \, d\mathbb{P} .$$

- $\mathbb{E}(X \mid \mathcal{G})$ always exits and satisfies $\mathbb{E}(W\, \mathbb{E}(X \mid \mathcal{G})) = \mathbb{E}(WX)$ $\forall\, W : \Omega \to \mathbb{R}$, $\mathcal{G}$-measurable and bounded.

$\Rightarrow$ $\mathbb{E}(\mathbb{E}(X \mid \mathcal{G})) = \mathbb{E}(X)$.

\+ Various properties, quite natural.