

Reminder IV

- Stochastic process $X = (\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{t \in \mathbb{T}}, (X_t)_{t \in \mathbb{T}})$.
prob space, filtration, random variables adapted to filtration.
- Natural filtration, augmented natural filtration.
- Modifications of X , indistinguishable stochastic processes.
- Progressively measurable stochastic process \Leftarrow
continuous stochastic process ($\exists t \mapsto X_t(\omega) \in \mathbb{R}$
continuous). Almost surely continuous sto. pro. standard ↓
- Right continuity of filtration.
- 1D Brownian motion $(\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{t \in \mathbb{R}_+}, (B_t)_{t \in \mathbb{R}_+})$
with $B_0 = 0$, $B_t - B_s$ independent of $\mathcal{F}_s \forall 0 \leq s \leq t$,
 $B_t - B_s \sim N(0, t-s) \forall 0 \leq s < t$.
- Continuity ✓ Gaussian process ✓ $\mathbb{E}(B_s B_t) = t \wedge s$ ✓
 \leadsto other characterization of 1D B.m with augmented
natural right continuous filtration.
- Properties under transformations.