

# Reminder XIII

- Put/call European / American options

$K =$  exercise price,  $T =$  maturity time,

$Z =$  payoff distribution ( $(S_T - K)^+$  for European call).  
price process  
reference, risk free asset

- Market model :  $(S^0, S^1, \dots, S^n)$  stochastic processes

with values in  $\mathbb{R}_+$  + SDE. Example : B-S model.

- Trading strategy  $H : [0, T] \times \Omega \rightarrow \mathbb{R}^{n+1}$

$V_t(H) := \sum_{i=0}^n H_t^i S_t^i$  portfolio.

arbitrage free market

- Self-financing, admissible, arbitrage.

- Discounted price process, discounted portfolio.

- Equivalent martingale measure  $P^* \rightsquigarrow \tilde{S}^i$  a martingale

Prop :  $\exists P^* \Rightarrow$  arbitrage free model.

European option

$\tilde{V}(H)$  martingale under  $P^*$

- $(Z, T)$  is attainable if  $\exists H \in M_T(P^*)$  s.t.  $V_T(H) = Z$ .

$$V_t(H) = E^* \left( e^{-\int_0^t r_s ds} Z \mid \mathcal{F}_t \right)$$

right price for  $(Z, T)$  at time  $t$ , otherwise arbitrage.