Reminder XII

- $U_t f := \int_{\mathbb{R}} f(z)\, p(t, \cdot, dz)$ ⇝ Contraction semi group, infinitesimal generator A. ↖ homogeneous case

in-homogeneous case ↙ $U_{s,t} f := \int_{\mathbb{R}} f(z)\, p(s, t, \cdot, dz)$, satisfies

$U_{s,t} = U_{s,u} U_{u,t}$ for $t > u > s \geq 0$. Generators A_s^+, A_s^-.

- For $X_t = \sigma(t, X_t)\, dB_t + \mu(t, X_t)\, dt$ time in-homogeneous diffusion process

↗ SDE

set $L_t := \frac{1}{2} \sigma(t,x)^2 \partial_x^2 + \mu(t,x)\partial_x$ 2nd order differential operator

- Backward equation with <u>terminal value</u> : $f \in C^{1,2}(\mathbb{R}_+ \times \mathbb{R})$,

PDE ↙

sol. of $(\partial_t + L_t) f = 0$, $f(T,y) = g(y)$, then for $0 \leq t < T$

solution PDE ↘ ↙ expectation of SDE

$f(t,y) = \mathbb{E}(g(X_T) \mid X_t = y)$. Converse holds Kolmogorov's equation

- Backward equation with <u>initial value</u> : $\partial_t f = L_t f$,

$f(0,y) = g(y)$, then $f(t,y) = \mathbb{E}(g(X_t) \mid X_0 = y)$, $t > 0$.

✓ time homogeneouse ↙ time in-homogeneous

- Infinitesimal generator $A = L$, or $A_t = L_t$.

- For $(\partial_t + L_t) f = r f$, terminal value ⇝ Feynman Kac formula

- Formal adjoint L_t^* of L_t ⇝ backward equation

induced density function ↙

or Fokker-Planck equation : $\partial_t \mu_t = L_t^* \mu_t$ or

$\partial_t p = L_t^* p$. ↖ Markov transition density.