

Reminder X

• Itô process : $X_t = Y + \int_0^t V_s dB_s + \int_0^t D_s ds$ *

with $V \in M_{loc}^2([0, T])$, $D \in M_{loc}^1([0, T])$, + adapted

* $\Leftrightarrow dX_t = V_t dB_t + D_t dt$, $X_0 = Y$.

• Itô's lemma for Itô processes $f(t, X_t) = \dots$

Integrals with respect to X_t , instead of B_t .

• Extension to more than 1 Itô process, but 1 B!
different B is also possible...

• Deterministic differential equations :

$f(t, x(t), x'(t), \dots, x^{(m)}(t)) = 0$, find $x: \mathbb{R} \rightarrow \mathbb{R}$?

Special case : $x'(t) = f(t, x(t)) \Leftrightarrow dx(t) = f(t, x(t)) dt$

• Stochastic differential equations (SDE) : special 1st order SDE

$dX_t = \underbrace{f(t, X_t) dt}_{\in M_{loc}^1([0, T])?} + \underbrace{\sigma(t, X_t) dB_t}_{\in M_{loc}^2([0, T])?}$, $X_0 = Y$

• Strong solutions : everything well defined, adapted to B.

• Explicit solution for linear SDE. ✓

• Sufficient conditions for existence and uniqueness. ✓
↳ regularity + growth conditions on f, σ . + condition on Y .