Reminder I

- Measurable space, measurable function.
- Probability space $(\Omega, \mathcal{F}, \mathbb{P})$ probability measure

sample space event space

- Random variable : $X : \Omega \longrightarrow \Lambda$

$\int (\Omega, \mathcal{F}, \mathbb{P})$ $\sim (\Lambda, \mathcal{E})$

Induced probability measure $\mu_X(A) = \mathbb{P}(X^{-1}(A))$ $\in \mathcal{E}$

$= \mathbb{P}(\{\omega \in \Omega \mid X(\omega) \in A\})$

$\Rightarrow \mu_X : \mathcal{E} \longrightarrow [0,1]$

- If $\Lambda = \mathbb{R}$, X univariate r.v.
  $\Lambda = \mathbb{R}^N$, X multivariate r.v., or random vector
  $\Lambda = \mathbb{R}^N$ and $\exists\ \Pi_X : \mathbb{R}^N \to \mathbb{R}_+$ with

  $\mu_X(A) = \int_A \Pi_X(x)\, dx$ X absolutely continuous

  pdf

  $X(\Omega)$ finite or countable, then $p_X(x) := \mathbb{P}(X^{-1}(\{x\}))$
  and X is discrete valued pmf

- If only Π_X or p_X are given : probability distribution

- If $X : \Omega \to \Lambda$ and $f : \Lambda \longrightarrow \Xi$ $(\Xi, \mathcal{G})$ standard

then $\mathbb{E}(f(X)) := \int_\Lambda f(x)\, \mu_X(dx)$

if it converges absolutely $\mathbb{E}$ = expectation