# Correspondence Between Definition 3.18 and the $\epsilon - \delta$ Definition of Continuity

FIRDAUS Rafi Rizqy / 062101889

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**Exercise 3.1.9**

*Let $M = N = \mathbb{R}$ with the usual topology defined by open sets. Check that the notion of continuity introduced in **Definition 3.1.8** corresponds to the standard definition of a continuous function $f : \mathbb{R} \to \mathbb{R}$ in terms of $\epsilon$ and $\delta$.*

**Proof:** Given $x_0 \in \mathbb{R}$ and given $\epsilon > 0$, the interval $U = (f(x_0) - \epsilon, f(x_0) + \epsilon)$ is an open set of the range space $\mathbb{R}$. We know that $f^{-1}(U)$ is open and contains the point $x_0$ such that we can construct a basis element $(a, b)$ about the point $x_0$. Therefore, we can choose $\delta$ to be smaller than the two numbers $x_0 - a$ and $b - x_0$. Then if $|x - x_0| < \delta$, the point $x$ must be in the interval $(a, b)$ such that we have $f(x) \in U$. Hence, for any $\epsilon > 0$ there exists $\delta > 0$ such that

$$|f(x) - f(x_0)| < \epsilon, \ \ \forall x \text{ satisfying } |x - x_0| < \delta.$$

Therefore, the notion of continuity introduced in **Definition 3.1.8** implies the $\epsilon - \delta$ definition of continuity.