Reminder IX

• Distance on $GL(n, \mathbb{K})$: $d(A,B) := \left(\sum_{j,k=1}^{n} |a_{jk} - b_{jk}|^2\right)^{1/2}$ ($\mathbb{K}$: $\mathbb{R}$ or $\mathbb{C}$)

$\rightsquigarrow$ topology on $GL(n, \mathbb{K})$, from balls $B(A, r)$.

• Linear Lie groups : closed subgroups of $GL(n, \mathbb{K})$, with identity denoted by $\mathbb{1}$ instead of e.

• Euclidean group, Poincaré group, matrices groups $\Big\} \in \{$linear Lie groups$\} \subset \{$Lie groups$\}$
↑ Cartan's theorem

• If (V, φ) is a local coordinates system : $\varphi : V \to \mathbb{R}^d$, ($V \subset G$, local homeomorphism)

then $\varphi^{-1} : \varphi(V) \to G \subset GL(n, \mathbb{K}) \subset \mathbb{R}^{n^2 \text{ or } 2n^2}$ is smooth. ($\varphi(V) \subset \mathbb{R}^d$)

• $X_\ell := \lim_{t \to 0} \dfrac{\varphi^{-1}(tE_\ell) - \mathbb{1}}{t} \in M_n(\mathbb{K})$, and $\{X_\ell\}_{\ell=1}^{d}$, with the commutator of matrices, is a real Lie algebra. The tangent space is a Lie algebra of dimension d, with suitable operation.

• For $X \in L(G)$, $\mathbb{R} \ni t \mapsto \exp(tX) \in G_0 \subset G$ is a 1 parameter family, with $\frac{d}{dt} \exp(tX)\big|_{t=0} = X$ (generator)

• Any $G_0 \ni A = \exp(X_1) \dots \exp(X_N)$, $X_j \in L(G)$.

$N = 1$ if G compact.

• Campbell - Backer - Hausdorff formula about $\exp(X)\exp(Y) = \dots$