Reminder VIII

• <u>Topological manifold</u> : Hausdorff second countable topological space $(\mathcal{M}, \mathcal{T})$, locally homeomorph to open subsets of $\mathbb{R}^n$.

For any $\varphi : V \to \mathbb{R}^n$, $\varphi(q) := (x_1(q), x_2(q), \dots, x_n(q)) \in \mathbb{R}^n$ ($\in \mathcal{M}$)

provides a <u>local coordinate system</u> / <u>local parametrization</u> by n parameters.

transition function

• <u>Smooth manifold</u> : Top. manifold with $\varphi_k \circ \varphi_j^{-1}$ smooth.

• <u>Lie group</u> : group + smooth manifold with group law and inversion smooth.

• <u>Compact</u> topological space, compact subset.

• <u>Integration</u> on Lie group $\rightsquigarrow$ <u>Haar measure</u>.

If G compact, $\int_G 1 \; \mu(da) = 1$ ← normalization

$\rightsquigarrow$ all results of finite groups extend to compact groups, if we consider <u>strongly continuous</u> representations.

- Connected, path-connected, simply connected top. space.
- Identity component G_0 is normal subgroup.
- Lie algebra (over $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$): $\mathbb{K}$-vector space L

with Lie bracket $[\cdot,\cdot] : L \times L \to L$ satisfying

1) $[\alpha X + \beta Y, Z] = \alpha [X, Z] + \beta [Y, Z]$, linearity

2) $[X, Y] = -[Y, X]$, anti-commutativity

Jacobi identity

3) $[X,[Y,Z]] + [Y,[Z,X]] + [Z,[X,Y]] = 0$.

- If $\{X_1, \dots, X_n\}$ is a basis of L, $[X_j, X_k] = \sum_{p=1}^{n} C_{jk}^{p} X_p$.

structure coefficients