Reminder III

- Transformation group : $\circ : G \times X \longrightarrow X$ s.t.

$b \circ (a \circ x) = (ba) \circ x$ and $e \circ x = x$.

- Euclidean group $E(n)$: recall $d^2(x,y) = \langle x-y, x-y \rangle = \|x-y\|^2$.

$E(n)$ preserves the distance d : $d(a \circ x, a \circ y) = d(x,y)$

$a \equiv (b, B)$ with $b \in T(n) \equiv (\mathbb{R}^n, +)$, $B \in O(n)$, and

$(b, B) \circ x = Bx + b$. Then $E(n) = \underset{\text{translation}}{T(n)} \rtimes \underset{\text{rotation}}{R(n)}$

- Lorentz group $\mathcal{L}$: recall $x \cdot y = x_0 y_0 - x_1 y_1 - x_2 y_2 - x_3 y_3$.

$\mathcal{L}$ preserves $\cdot$: $\mathcal{L} = \{ \Lambda \in M_4(\mathbb{R}) \mid \Lambda x \cdot \Lambda y = x \cdot y \}$.

- Poincaré group $\mathcal{P}$: recall $t(x,y) = (x-y) \cdot (x-y)$.

$\mathcal{P}$ preserves t : $t(a \circ x, a \circ y) = t(x,y)$

$a \equiv (b, \Lambda)$ with $b \in T(4)$, $\Lambda \in \mathcal{L}$ and

$(b, \Lambda) \circ x = \Lambda x + b$. Then $\mathcal{P} = T(4) \rtimes \mathcal{L}$.

- <u>Vector space</u> $\mathcal{V}$ and <u>Hilbert space</u> $\mathcal{H}$

→ define on $\mathbb{C}$, can be infinite dimensional

vector space + <u>scalar product</u> $\langle \cdot, \cdot \rangle$

⇝ <u>norm</u> $\| \cdot \|$.

- <u>Linear map / operator</u>: $T(f+\lambda g) = Tf + \lambda Tg$

$\forall f, g \in \mathcal{V}, \ \lambda \in \mathbb{C}$

Set of all linear maps: $\mathcal{L}(\mathcal{V})$.

- For Hilbert spaces:

$\underline{\mathcal{B}(\mathcal{H})} := \{ T \in \mathcal{L}(\mathcal{H}) \mid \|Tf\| \leq c\|f\|$, for some $c \geq 0$

and all $f \in \mathcal{H} \}$

The inf on all c is denoted by $\|T\|$ ⚠

- <u>adjoint</u> $\langle f, Tg \rangle = \langle T^* f, g \rangle$

<u>unitary</u> if $T^*T = TT^* = \mathbb{1}$ $\Leftrightarrow$ $T^* = T^{-1}$

<u>invertible</u> if bijective, inverse T^{-1}

<u>Remark</u>: if $\dim(\mathcal{V}) < \infty$, then $\mathcal{V} \simeq \mathbb{C}^n$ for some n.

has a scalar product