Reminder II

- $\phi : G \to G'$, $\phi(ab) = \phi(a)\phi(b)$ homomorphism, isomorphism endomorphism, automorphism

Always : $\phi(e_G) = e_{G'}$ and $Ker(\phi)$ normal subgroup.

Example : $\phi : SU(2) \to SO(3)$, $Ker(\phi) = \{-1\!\!1, 1\!\!1\}$

double - cover

- Direct product , semi-direct product

$G_1 \times G_2$ (easy) $N \rtimes H$ (more intricate)

outer : create a new group from 2 groups

inner : detect the special structure of a group.

- Transformation group = group G acting on a set X.

$\circ : G \times X \to X$ with $e \circ x = x$, $a \circ (b \circ x) = (ab) \circ x$.

Orbit $O_x \subset X$, Stabilizer $G_x \subset G$.

Examples :

1) translation group $T(n) \simeq (\mathbb{R}^n, +)$ $X = \mathbb{R}^n$, preserves distance $d(x,y) = \|x - y\|$

2) rotation group $R(n) \simeq O(n)$ $X = \mathbb{R}^n$, preserves $\langle \cdot , \cdot \rangle$

3) Euclidean group $E(n)$ $X = \mathbb{R}^n$, preserves distance

general notation

$a \equiv (a, B)$ with $a \in T(n)$, $B \in O(n)$ and

$(a, B) \circ x := Bx + a$.