Nagoya University, G30 program Fall 2022

Mathematics Tutorial Ia Instructor : Serge Richard

Homework 7

Exercise 1 *Prove the following properties of the function* ln *:*

(i) $\ln(x)' = \frac{1}{x}$ *for any* $x \in (0, \infty)$,

(ii) $\ln(xy) = \ln(x) + \ln(y)$ *for any* $x, y \in (0, \infty)$,

(iii) $\ln(x^q) = q \ln(x)$ *for any* $x \in (0, \infty)$ *and* $q \in \mathbb{Q}$.

Exercise 2 *Let us set* $\varepsilon := \mathrm{e}^1 = 2.718....$ *Check that* $\ln(\varepsilon) = 1$ *and that* $\varepsilon^x = \mathrm{e}^x$.

Exercise 3 *Compute the derivative of the following functions:*

$$f : \mathbb{R} \ni x \mapsto a^x \in \mathbb{R} \ \text{ for any } a > 0, \qquad g : \mathbb{R}_+^* \ni x \mapsto x^x \in \mathbb{R}.$$

Exercise 4 *Compute the following limits:*

$$a) \lim_{x \to 0_+} x \ln(x), \qquad b) \lim_{x \to 0_+} x^x, \qquad c) \lim_{x \to +\infty} \frac{\ln(x)}{x}, \qquad d) \lim_{x \to +\infty} x^{1/x} .$$

What can you say for $\lim_{x \to 0_+} x^r \ln(x)$ *for any* $r > 0$ *?*

Exercise 5 *Compute the following limits:*

$$a) \lim_{x \to 0} \frac{\ln(1+x)}{x}, \quad b) \lim_{x \to 0} (1+x)^{1/x}, \quad c) \lim_{x \to \infty} \big(1 + \frac{1}{x}\big)^x, \quad d) \lim_{x \to \infty} \big(1 + \frac{r}{x}\big)^x \text{ for any } r > 0.$$