

Homework 2

Exercise 1 Consider the expression $\frac{x^2+3x+2}{x^2+x-2}$. Determine for which $x \in \mathbb{R}$ this expression is well-defined, and simplify this expression. Determine the maximal domain on which the function $x \mapsto \frac{x^2+3x+2}{x^2+x-2}$ can be defined.

Exercise 2 A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is said to be even if $f(-x) = f(x)$ for any $x \in \mathbb{R}$, and f is said to be odd if $f(-x) = -f(x)$ for any $x \in \mathbb{R}$.

1) Determine which of the functions defined for $x \in \mathbb{R}$ by

$$a) f(x) = x, \quad b) f(x) = x^2, \quad c) f(x) = x^2 + x, \quad d) f(x) = \sin(x), \quad e) f(x) = \begin{cases} 1/x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

are even or odd ?

2) Show that for any function f , the function $g : \mathbb{R} \rightarrow \mathbb{R}$ defined by $g(x) = \frac{1}{2}(f(x) + f(-x))$ is an even function while the function $h : \mathbb{R} \rightarrow \mathbb{R}$ defined by $h(x) = \frac{1}{2}(f(x) - f(-x))$ is an odd function. In addition, observe that $f = g + h$.

3) What can you say about the graph of an even function, and about the graph of an odd function ?

Exercise 3 Consider $f : \mathbb{R} \ni x \mapsto x^2 \in \mathbb{R}$ and $g : \mathbb{R}_+ \ni x \mapsto \sqrt{x} - 1 \in \mathbb{R}$, and determine if the following functions are well defined :

$$a) f \circ g, \quad b) f \circ f, \quad c) g \circ g.$$

If one function is not well defined, on which maximal domain can one define it ?

Exercise 4 Sketch the graph of the following function $f : \mathbb{R} \rightarrow \mathbb{R}$ given for $x \in \mathbb{R}$ by

$$a) f(x) = \frac{1}{2}x - 2, \quad b) f(x) = x - 3, \quad c) f(x) = x^2 + 3x + 2, \quad d) f(x) = \begin{cases} 1 + 1/(x - 2) & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}.$$

Exercise 5 Determine the equation of the function $f : \mathbb{R} \rightarrow \mathbb{R}$ whose graph is a straight line containing the points (x_1, y_1) and (x_2, y_2) of \mathbb{R}^2 . What is the slope of this line ?

Exercise 6 Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be two continuous functions. Show as precisely as possible that

1. the sum $\lambda f + g$ is continuous on \mathbb{R} for any $\lambda \in \mathbb{R}$,
2. the product fg is continuous on \mathbb{R} .