Report by Li Yucheng

The solution to $\begin{cases} x(m+1) = A(m)x(m) + g(m) \\ x(m_0) = x_0 \end{cases}$ is

$$x(m) = \Pi(m, m_0)x_0 + \sum_{j=m_0}^{m-1} \Pi(m, j+1)g(j).$$

Proof:

We just need to take this solution into the equation and have a check. If the solution is right, then

$$\begin{aligned} x(m+1) &= \Pi(m+1, m_0)x_0 + \sum_{j=m_0}^{m} \Pi(m+1, j+1)g(j). \\ &= A(m)\Pi(m, m_0)x_0 + \sum_{j=m_0}^{m} \Pi(m+1, j+1)g(j) + \Pi(m+1, m+1)g(m) \\ &= A(m)\Pi(m, m_0)X_0 + A(m)\sum_{j=m_0}^{m-1}\Pi(m, j+1)g(j) + g(m) \\ &= A(m)\left[\Pi(m, m_0)x_0 + \sum_{j=m_0}^{m-1} \Pi(m, j+1)g(j)\right] + g(j) \\ &= A(m)x(m) + g(m) \end{aligned}$$

When $m = m_0$, $x(m_0) = \Pi(m_0, m_0)X_0 = X_0$, which corresponds to the boundary condition.