Report by Li Yucheng

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In the latest lecture, we have known that there are two stable fixed points $(\pm1, 0)$ and one unstable fixed point $(0,0)$ in the case $\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \begin{pmatrix}x(1-x^2)\\ -y\end{pmatrix}$.

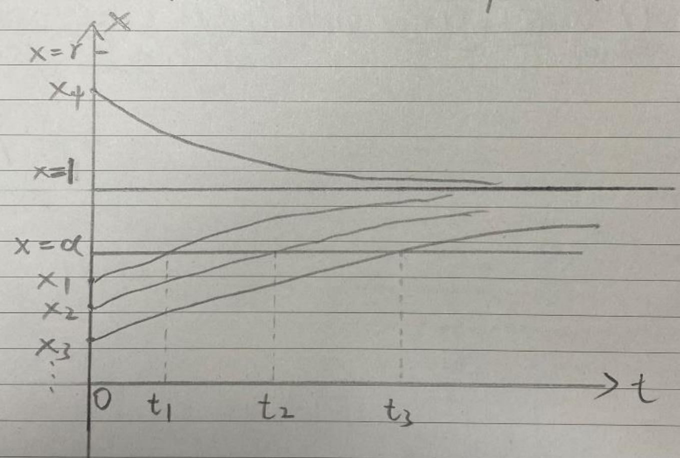
Then, we can say that $w_+(B_r(0)) = [-1,1]\times\{0\}, \forall r>1$ and $\bigcup_{x\in B_r(0)} w_+(x) = \{(-1,0), (0,0), (1,0)\}$

Proof:

① $w_+(B_r(0)) = [-1,1]\times\{0\}, \forall r>1$;

By $\dot{y}=-y$ we can easily see that every point in $B_r(0)$ will go to x axis when $t\to\infty$. So we just need to prove that $w_+(X) = [-1,1], X=[-r,r]$

The trajectories of x on t can be expressed in such a figure:



It is obvious that ±1 and $0 \in w_+(X)$, because when $x=\pm1$ or 0, $\dot{x}=0$, so it will stay at ±1 or 0 as $t\to\infty$.

For $\forall\alpha\in(0,1)$, we can construct such two sequences of x and t, with $x_n\to0$, as shown in this figure. When x_n is going to 0, the speed of x going toward 1 also becomes very small. Finally, when x_n is infinitly near 0 (but not at 0), the speed of x will be infinitly small so that it takes infinite time for x to reach α, which means $t_n\to\infty$. So, $\exists t_n\to\infty$ and $x_n\in X$ with $\phi(t_n,x_n)\to\alpha$. By definition, $\alpha\in W_+(X)$ and then $[0,1]\in W_+(X)$.

Then we need to prove that $\beta \notin w_+(X)$ if $\beta > 1$. In fact, when $x > 1$, the speed $\dot{x}$ goes infinitely small only when $x \to 1^+$. Thus, it takes infinite time for x_n to reach β (otherwise there does not exist a $t_n \to \infty$) only when $\beta = 1$ and this contradicts to $\beta > 1$. Since $\dot{x} = x(1-x^2)$ is symmetrical on the sign of x, we can conclude that $[-1,1] \in W_+(X)$ and $\beta \notin w_+(X)$ if $|\beta| > 1$. Thus, $W_+(X) = [-1,1]$, then $W_+(B_r(0)) = [-1,1] \times \{0\}$, $\forall r > 1$.

② $\bigcup_{x \in B_r(0)} w_+(x) = \{(-1,0), (0,0), (1,0)\}$:

Since this is a union of $w_+(x)$, x is fixed. So we cannot construct a sequence x_n as in the first part of proof. Then by definition of $w_+(x)$ and Professer Serge's note, it is easy to get that $\bigcup_{x \in B_r(0)} w_+(x) = \{(-1,0), (0,0), (1,0)\}$.

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