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Proof of invariance of U ∩ V, U ∪ V, U/V and closure of U

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No.

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If U, V are invariant, then $U\cap V$, $U\cup V$, $U\setminus V$ and $\bar{U}$ are invariant.

Proof:

From the definition of invariant, we can know that $\gamma(x)\subset U(V)$, $\forall x\in U(V)$.

$U\cap V$: Suppose $x\in U\cap V$. $\because$ $\gamma(x)\subset U$ and $\gamma(x)\subset V$.
$\therefore \gamma(x)\subset U\cap V$

$U\cup V$: Suppose $x\in U\cup V$. Then x is either in U or V (or both).
$\therefore \gamma(x)\subset U$ or V (or both)
$\therefore \gamma(x)\subset U\cup V$

$U\setminus V$: We can make a contradiction. Suppose $x\in U/V$ and part of $\gamma(x)$ is in V. Suppose y is on that part.
Thus $\gamma(x)=\gamma(y)$
$\because y\in V$ $\therefore \gamma(y)\in V$, $\gamma(x)=\gamma(y)\in V$
This contradicts to the fact that $x\in U/V$
$\because U$ is also invariant, $\gamma(x)$ can not go outside of U
$\therefore U/V$ is invariant.

$\bar{U}$: Since $\phi(t,x)$ is continuous on x,
$\forall \varepsilon>0, \exists \delta>0$ s.t $|x-y|<\delta$, $|\phi(t,x)-\phi(t,y)|<\varepsilon$.
Thus we have
$\lim_{y\to x}\phi(t,y)=\phi(t,\lim_{y\to x}y)=\phi(t,x)$.

Suppose x is a point on the boundary of U. and x_n is a sequence which converges to x. Then
$\phi(t,x_n)\in U$ for every n.
$\because \phi(t,x)=\phi(t,\lim_{n\to\infty}x_n)=\lim_{n\to\infty}\phi(t,x_n)$
$\therefore \phi(t,x)\in\bar{U}$ (otherwise it would contradict to the continuity)
$\therefore \bar{U}$ is invariant.