

**Homework 8**

**Exercise 1** Compute the following limits:

$$a) \lim_{x \rightarrow 0} \frac{\ln(1+x)}{x}, \quad b) \lim_{x \rightarrow 0} (1+x)^{1/x}, \quad c) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x, \quad d) \lim_{x \rightarrow \infty} \left(1 + \frac{r}{x}\right)^x \text{ for any } r > 0.$$

**Exercise 2** Compute

$$a) \lim_{x \rightarrow 1} \left( \frac{x}{x-1} - \frac{1}{\ln(x)} \right),$$

$$b) \lim_{x \rightarrow 0^+} (1 + \sin(4x))^{\cot(x)} \quad \text{with } \cot(x) = \frac{1}{\tan(x)}.$$

**Exercise 3** Consider the function  $\tanh : \mathbb{R} \rightarrow (-1, 1)$ . Show that this function is invertible and compute the derivative of its inverse.

**Exercise 4** Show that  $\tanh(y)^{-1} = \frac{1}{2} \ln \left( \frac{1+y}{1-y} \right)$  for any  $y \in (-1, 1)$ .

**Exercise 5** Differentiate the function  $\mathbb{R}_+ \ni x \mapsto \frac{x^{3/4} \sqrt{x^2+1}}{(3x+2)^5} \in \mathbb{R}_+$ .