

**Homework 7**

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**Exercise 1** Prove the following properties of the function  $\ln$  :

(i)  $\ln(x)' = \frac{1}{x}$  for any  $x \in (0, \infty)$ ,

(ii)  $\ln(xy) = \ln(x) + \ln(y)$  for any  $x, y \in (0, \infty)$ ,

(iii)  $\ln(x^q) = q \ln(x)$  for any  $x \in (0, \infty)$  and  $q \in \mathbb{Q}$ .

**Exercise 2** Let us set  $e := e^1 = 2.718\dots$ . Check that  $\ln(e) = 1$  and that  $e^x = e^x$ .**Exercise 3** Compute the derivative of the following functions:

$$f : \mathbb{R} \ni x \mapsto a^x \in \mathbb{R} \text{ for any } a > 0, \quad g : \mathbb{R}_+^* \ni x \mapsto x^x \in \mathbb{R}.$$

**Exercise 4** Compute the following limits:

$$a) \lim_{x \rightarrow 0_+} x \ln(x), \quad b) \lim_{x \rightarrow 0_+} x^x, \quad c) \lim_{x \rightarrow +\infty} \frac{\ln(x)}{x}, \quad d) \lim_{x \rightarrow +\infty} x^{1/x}.$$

What can you say for  $\lim_{x \rightarrow 0_+} x^r \ln(x)$  for any  $r > 0$  ?