

Homework 8**Exercise 1** Compute

a) $\lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\ln(x)} \right),$

b) $\lim_{x \rightarrow 0_+} (1 + \sin(4x))^{\cot(x)} \quad \text{with } \cot(x) = \frac{1}{\tan(x)}.$

Exercise 2 Compute the following limits:

a) $\lim_{x \rightarrow 0_+} x \ln(x),$ b) $\lim_{x \rightarrow 0_+} x^x,$ c) $\lim_{x \rightarrow +\infty} \frac{\ln(x)}{x},$ d) $\lim_{x \rightarrow +\infty} x^{1/x}.$

What can you say for $\lim_{x \rightarrow 0_+} x^r \ln(x)$ for any $r > 0$?**Exercise 3** Compute the following limits:

a) $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x},$ b) $\lim_{x \rightarrow 0} (1+x)^{1/x},$ c) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x,$ d) $\lim_{x \rightarrow \infty} \left(1 + \frac{r}{x}\right)^x$ for any $r > 0.$

Exercise 4 Consider the function $\tanh : \mathbb{R} \rightarrow (-1, 1).$ Show that this function is invertible and compute the derivative of its inverse.**Exercise 5** Show that $\tanh(y)^{-1} = \frac{1}{2} \ln \left(\frac{1+y}{1-y} \right)$ for any $y \in (-1, 1).$