Nagoya University, G30 program Fall 2020
Mathematics Tutorial Ia ❤ Instructor : Serge Richard

Homework 7

Exercise 1 *Prove the following properties of the function* ln *:*

(i) $\ln(x)' = \frac{1}{x}$ *for any* $x \in (0, \infty)$,

(ii) $\ln(xy) = \ln(x) + \ln(y)$ *for any* $x, y \in (0, \infty)$,

(iii) $\ln(x^q) = q \ln(x)$ *for any* $x \in (0, \infty)$ *and* $q \in \mathbb{Q}$.

Exercise 2 *Let us set* $\varepsilon := \mathrm{e}^1 = 2.718\ldots$. *Check that* $\ln(\varepsilon) = 1$ *and that* $\varepsilon^x = \mathrm{e}^x$.

Exercise 3 *Compute the derivative of the following functions:*

$$f : \mathbb{R} \ni x \mapsto a^x \in \mathbb{R} \ \text{ for any } a > 0, \qquad g : \mathbb{R}_+^* \ni x \mapsto x^x \in \mathbb{R}.$$

Exercise 4 *Differentiate the function* $\mathbb{R}_+ \ni x \mapsto \frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5} \in \mathbb{R}_+$.

Exercise 5 *Compute*
a) $\lim_{x\to 1} \left(\frac{x}{x-1} - \frac{1}{\ln(x)} \right)$,
b) $\lim_{x\to 0_+} \big(1 + \sin(4x)\big)^{\cot(x)}$ *with* $\cot(x) = \frac{1}{\tan(x)}$.