Nagoya University, G30 program Fall 2020

Mathematics Tutorial Ia ● ☽ ○ ☾ . . . ☼ Instructor : Serge Richard

---

Homework 6

Exercise 1 *Compute the following limits:*

(i) $\lim_{x\to 0} \frac{x-\sin(x)}{x^3}$,

(ii) $\lim_{x\to 0} \frac{x^2}{1+x-e^x}$.

Exercise 2 *Find the critical points for the differentiable functions* $f:\mathbb{R}\to\mathbb{R}$ *defined for* $x\in\mathbb{R}$ *by*

$$a)\ -x^2+2x+2, \qquad b)\ x^3-3, \qquad c)\ \cos(x), \qquad d)\ \sin(x)+\cos(x).$$

Exercise 3 *Find the point of the curve of equation* $y^2 = 4x$ *which is the nearest one to the point* $(2,3)$.

Exercise 4 *Show that* $\sin(x) \leq x$ *for any* $x \geq 0$.

Exercise 5 *Show that there are exactly two tangent lines to the graph of the function* $f:\mathbb{R}\ni x\mapsto (x+1)^2\in\mathbb{R}$ *which pass through the origin. Find the equation of these lines (*$\Leftrightarrow$ *find the two functions whose graphs correspond to these straight lines).*

Exercise 6 *Prove the following statement:* Let $f:[a,b]\to\mathbb{R}$ be a strictly increasing and continuous function, and set $\alpha := f(a)$ and $\beta := f(b)$. Then there exists an inverse function $f^{-1}:[\alpha,\beta]\to[a,b]$ such that $f^{-1}\big(f(x)\big) = x$ and $f\big(f^{-1}(y)\big) = y$ for any $x\in[a,b]$ and $y\in[\alpha,\beta]$.