Homework 4

Exercise 1 *Compute the derivative of the functions* $f : \mathbb{R} \to \mathbb{R}$ *with* $f(x)$ *provided by the following expressions:*

$$a)\ 5x^4 + 4x^2 - 1, \qquad b)\ (x^5+1)(x^2-1), \qquad c)\ \frac{5x-1}{x-5} \ \text{for } x \neq 5, \qquad d)\ \frac{x^{25}-2x}{x^2+3}.$$

Exercise 2 *By using that* $\sin'(x) = \cos(x)$ *show that* $\cos'(x) = -\sin(x)$ *for any* $x \in \mathbb{R}$.

Exercise 3 *Consider the function* $f : \mathbb{R} \to \mathbb{R}$ *defined by* $f(x) = x^2 \sin(1/x)$ *if* $x \neq 0$ *and* $f(0) = 0$.

1. *Show that* f *is continuous at* 0,
2. *Compute the derivative of* f *at* 0,
3. *Compute the derivative of* f *at any* $x \neq 0$,
4. *Show that the derivative of* f *is well-defined but that this derivative is not continuous at* 0.

Indication: *you can use that* $\lim_{y\to 0} \frac{\sin(y)}{y} = 1$.

Exercise 4 *a) Let* $f(x) = x^2 \sin(1/x)$ *and* $g(x) = \sin(x)$ *for any* $x \in (-1,0) \cup (0,1)$. *Show that* $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$ *does not exist, but that* $\lim_{x\to 0} \frac{f(x)}{g(x)} = 0$.
b) Explain how this example fits in with L'Hospital's rule ?

Exercise 5 *By using the indication mentioned above, compute the following limits:*

1. $\lim_{h\to 0} \frac{\cos(h)-1}{h}$,
2. $\lim_{h\to 0} \frac{\cos(h)-1}{h^2}$.

Exercise 6 *Find the equation of the tangent of the curve in* $\mathbb{R}^2$ *defined by the relation*

$$F(x,y) = x^2 - y^2 + 3xy + 12 = 0$$

at the point $(-4, 2)$.