
Homework 13

Exercise 1 *Provide the Taylor's expansion (for x around 0) of order 3 for the function*

$$(-1,1) \ni x \mapsto \ln\Big(\frac{1+x}{1-x}\Big) \in \mathbb{R}.$$

Can you find out several approaches for this computation? Sketch these approaches.

Exercise 2 *a) For any $x \in \mathbb{R}$ with $x \neq -1$ we consider the sequence $(a_n)_{n\in\mathbb{N}}$ given by*

$$a_n := \frac{x^n}{1+x^n}.$$

For which x does the limit $\lim_{n\to\infty} a_n$ exists ? Give the value of this limit whenever it exists. Represent your findings on a graph (the horizontal axis corresponds to the x-variable).

b) For which x the sequence of a_n defines a convergent series?

Exercise 3 *Consider $f : [a,b] \to \mathbb{R}$ continuous, and differentiable on (a,b), and suppose that f' is also continuous on $[a,b]$. Show that the length ℓ of the curve defined by $\big\{\big(x, f(x)\big) \mid x \in [a,b]\big\}$ is given by the expression*

$$\ell = \int_a^b \sqrt{1+f'(x)^2}\ \mathrm{d}x\ .$$