Homework 6

Exercise 1 *By using that* $\sin'(x) = \cos(x)$ *show that* $\cos'(x) = -\sin(x)$ *for any* $x \in \mathbb{R}$.

Exercise 2 *1) Compute the derivative of the function* $(0, \infty) \ni x \mapsto \frac{1}{x} \in (0, \infty)$.

2) For any $q \in \mathbb{Q}$ *with* $q > 0$ *let us define the function* P_{-q} *by* $P_{-q}(x) \equiv x^{-q} := \frac{1}{x^q}$. *With the previous statement together with the content of Exercise 4 of Homework 4, show that*

$$P'_{-q}(x) = -qx^{-q-1} \equiv -q\frac{1}{x^{q+1}}.$$

Exercise 3 *Show that* $\sin(x) \leq x$ *for any* $x \geq 0$.

Exercise 4 *Let us set* $e^{-x} := \frac{1}{e^x}$, *and consider the functions hyperbolic cosine* $\cosh : \mathbb{R} \to \mathbb{R}$ *and hyperbolic sine* $\sinh : \mathbb{R} \to \mathbb{R}$ *defined by the formulas*

$$\cosh(x) := \frac{e^x + e^{-x}}{2} \qquad \text{and} \qquad \sinh(x) := \frac{e^x - e^{-x}}{2}.$$

Compute the derivative of these functions and sketch the graph of the functions cosh *and* sinh. *Prove the following relation:*

$$\cosh(x)^2 - \sinh(x)^2 = 1, \qquad \forall x \in \mathbb{R}.$$

Exercise 5 *Compute and simplify the derivative of the functions* $f : \mathbb{R} \to \mathbb{R}$ *defined for* $x \in \mathbb{R}$ *by*

$$a)\ \sin\big((2x^2 - 3)^2\big) \qquad b)\ \frac{(x+3)^3}{(2x-3)^2 + 1} \qquad c)\ \frac{1}{\sin^2(3x) + 1}$$

Exercise 6 *Compute the derivatives of order* 1, 2 *and* 3 *for the functions* $f : \mathbb{R} \to \mathbb{R}$ *defined for* $x \in \mathbb{R}$ *by*

$$a)\ \cos(x) \qquad b)\ \cos(x)\sin(x) \qquad c)\ x^4 + x^3 + x^2 + x^1 + 1$$