Nagoya University, G30 program Fall 2019

Mathematics Tutorial Ia (calculus) Instructor : Serge Richard

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Homework 2

Exercise 1 *Consider the sequences* $(a_n)_{n\in\mathbb{N}^*}$ *defined below and show (with* ε *and* N*) that these sequences are convergent. Can you find their limit ?*

(i) $a_n = \frac{1}{n^2}$,

(ii) $a_n = \sqrt{n+1} - \sqrt{n}$.

(iii) $a_n = \sqrt{n^2+5n} - n$.

More challenging (and optional): *Consider* $a_n = \big(1+\frac{1}{n}\big)^n$ *and show that the corresponding sequence is convergent. In your proof you can use the equality*

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

with $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Exercise 2 *Show that the sequence* $(a_n)_{n\in\mathbb{N}}$ *given by* $a_1 = 1$ *and* $a_{n+1} = 3 - \frac{1}{a_n}$ *for any* $n \geq 1$ *is increasing and bounded from above by* 3*. Deduce that this sequence is convergent and give its limit.*

Exercise 3 *Consider two real sequences* $(a_n)_{n\in\mathbb{N}}$ *and* $(b_n)_{n\in\mathbb{N}}$ *such that* $\lim_{n\to\infty} a_n = 0$ *and* $|b_n| \leq C$ *for one* $C > 0$ *and all* $n \in \mathbb{N}$ *(we say that the sequence* $(b_n)_{n\in\mathbb{N}}$ *is bounded). Show that* $\lim_{n\to\infty} a_n b_n = 0$.

A *parametric curve on* $\mathbb{R}^2$ is a map

$$I \ni t \mapsto \big(x(t), y(t)\big) \in \mathbb{R}^2$$

where I is an interval of $\mathbb{R}$, and where $x : I \to \mathbb{R}$ and $y : I \to \mathbb{R}$ are real functions defined on I.

Exercise 4 *Represent the following parametric curves:*

(i) $x(t) = \cos(t)$ *and* $y(t) = \sin(t)$ *for any* $t \in [0, 2\pi]$,

(ii) $x(t) = \mathrm{e}^t \cos(t)$ *and* $y(t) = \mathrm{e}^t \sin(t)$ *for any* $t \in \mathbb{R}$,

(iii) $x(t) = \sin(2t)$ *and* $y(t) = \sin(3t)$ *for any* t.