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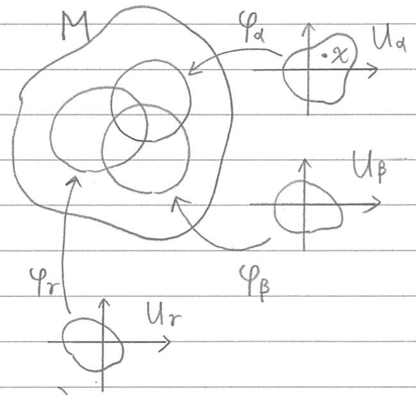
Uniqueness of the maximal atlas



Let (M, A) be a pair satisfying the condition (1), (2) (see the note), and S, T be maximal atlases induced by A.

We have to show $S = T$.

First let us show $S \subset T$.

Choose any $(U_\alpha, \varphi_\alpha) \in S$.

It suffices to prove that, for any $(U_\beta, \varphi_\beta) \in T$ with $\varphi_\alpha(U_\alpha) \cap \varphi_\beta(U_\beta) \neq \phi$, $\varphi_\alpha \circ \varphi_\beta^{-1}$ and $\varphi_\beta^{-1} \circ \varphi_\alpha$ are C^∞.

It suffices to prove that, for any $x \in \varphi_\alpha^{-1}(\varphi_\alpha(U_\alpha) \cap \varphi_\beta(U_\beta))$, $\varphi_\beta^{-1} \circ \varphi_\alpha$ is differentiable at x.

Let $x \in \varphi_\alpha^{-1}(\varphi_\alpha(U_\alpha) \cap \varphi_\beta(U_\beta))$ and choose $(U_\gamma, \varphi_\gamma) \in A$ s.t. $\varphi_\alpha(x) \in \varphi_\gamma(U_\gamma)$.

Then we can show that

$$\varphi_\beta^{-1} \circ \varphi_\alpha = (\varphi_\beta^{-1} \circ \varphi_\gamma) \circ (\varphi_\gamma^{-1} \circ \varphi_\alpha) \quad \text{on} \quad \varphi_\alpha(U_\alpha) \cap \varphi_\beta(U_\beta) \cap \varphi_\gamma(U_\gamma)$$

Since $\varphi_\beta^{-1} \circ \varphi_\gamma$ is C^∞ at $\varphi_\gamma^{-1} \circ \varphi_\alpha(x)$ ($\because T \supset A$) and $\varphi_\gamma^{-1} \circ \varphi_\alpha$ is C^∞ at x ($\because S \supset A$), it follows that

$$\varphi_\beta^{-1} \circ \varphi_\alpha \text{ is } C^\infty \text{ at } x.$$

Because $S \supset T$ is shown same as the argument,

$$S = T \quad ▨$$