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On the dimension of V^*

Suppose $\exists \{v_1, \dots, v_n\} \subset V$ $\quad n < \infty$
s.t $v \in V$
$$v = \sum_i \alpha_i v_i \quad \text{for some } \{a_i\}_{i=1,n}$$

i,e $\dim V = n$.

$$V^* = \{ v^* \mid v^*(v) \in \mathbb{R} \} \quad v \in V$$

define v_i^* s.t. $v_i^*(v_j) = \delta_{ij}$

$\Rightarrow \exists \{v_i^*\}_{i=\{1,n\}}$

Then for $\forall V^* \ni v^*$ specified by its action on $\{v_i\}_{i\in\{1,n\}}$
s.t
$$v^*(v_1) = c_1$$
$$\vdots$$
$$v^*(v_n) = c_n$$

$$v^* = \sum_{i=1}^n c_i v_i^*$$

and $\dim V^* = \dim V = n$.

Provided v_i^* are linearly independent.

Linear independence:

suppose $\exists \{\lambda_i\}_{i=1}^n$ s.t. $0 = \sum_{j=1}^n \lambda_j v_j^*$ $\quad 0 \in V^*$, with $|\lambda_k| > 0$ for some $k \in [1, \dots, n]$

$$\Rightarrow 0 = 0\left(\sum_{i=1}^n \bar\lambda_i v_i\right) = \sum_{j=1}\sum_{i=1} \bar\lambda_i \lambda_j v_j^*(v_i)$$
$$= \sum_{j=1}^n \sum_{i=1}^n \bar\lambda_i \lambda_j \delta_{ji}$$
$$= \sum_{i=1}^n |\lambda_i|^2$$
$$\geq |\lambda_k|^2 > 0$$

but $0 \not> 0$ so, by contradiction, $\{v_i^*\}$ are lin. indep.