
Homework 6

Exercise 1 By using that $\sin'(x) = \cos(x)$ show that $\cos'(x) = -\sin(x)$ for any $x \in \mathbb{R}$.

Exercise 2 1) Compute the derivative of the function $(0, \infty) \ni x \mapsto \frac{1}{x} \in (0, \infty)$.

2) For any $q \in \mathbb{Q}$ with $q > 0$ let us define the function P_{-q} by $P_{-q}(x) \equiv x^{-q} := \frac{1}{x^q}$. With the previous statement together with the content of Exercise 5 of Homework 4, show that

$$P'_{-q}(x) = -qx^{-q-1} \equiv -q\frac{1}{x^{q+1}}.$$

Exercise 3 Show that $\sin(x) \leq x$ for any $x \geq 0$.

Exercise 4 Let us set $e^{-x} := \frac{1}{e^x}$, and consider the functions hyperbolic cosine $\cosh : \mathbb{R} \rightarrow \mathbb{R}$ and hyperbolic sine $\sinh : \mathbb{R} \rightarrow \mathbb{R}$ defined by the formulas

$$\cosh(x) := \frac{e^x + e^{-x}}{2} \quad \text{and} \quad \sinh(x) := \frac{e^x - e^{-x}}{2}.$$

Compute the derivative of these functions and sketch the graph of the functions \cosh and \sinh . Prove the following relation:

$$\cosh(x)^2 - \sinh(x)^2 = 1, \quad \forall x \in \mathbb{R}.$$

Exercise 5 Compute and simplify the derivative of the functions $f : \mathbb{R} \rightarrow \mathbb{R}$ defined for $x \in \mathbb{R}$ by

$$\text{a) } \sin((2x^2 - 3)^2) \quad \text{b) } \frac{(x+3)^3}{(2x-3)^2 + 1} \quad \text{c) } \frac{1}{\sin^2(3x) + 1}$$

Exercise 6 Compute the derivatives of order 1, 2 and 3 for the functions $f : \mathbb{R} \rightarrow \mathbb{R}$ defined for $x \in \mathbb{R}$ by

$$\text{a) } \cos(x) \quad \text{b) } \cos(x)\sin(x) \quad \text{c) } x^4 + x^3 + x^2 + x^1 + 1$$