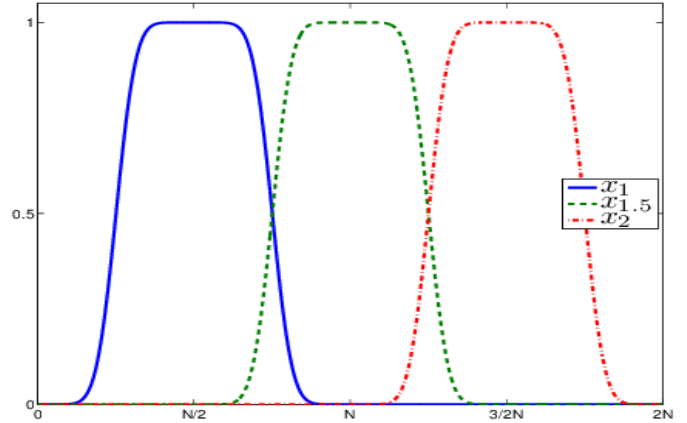
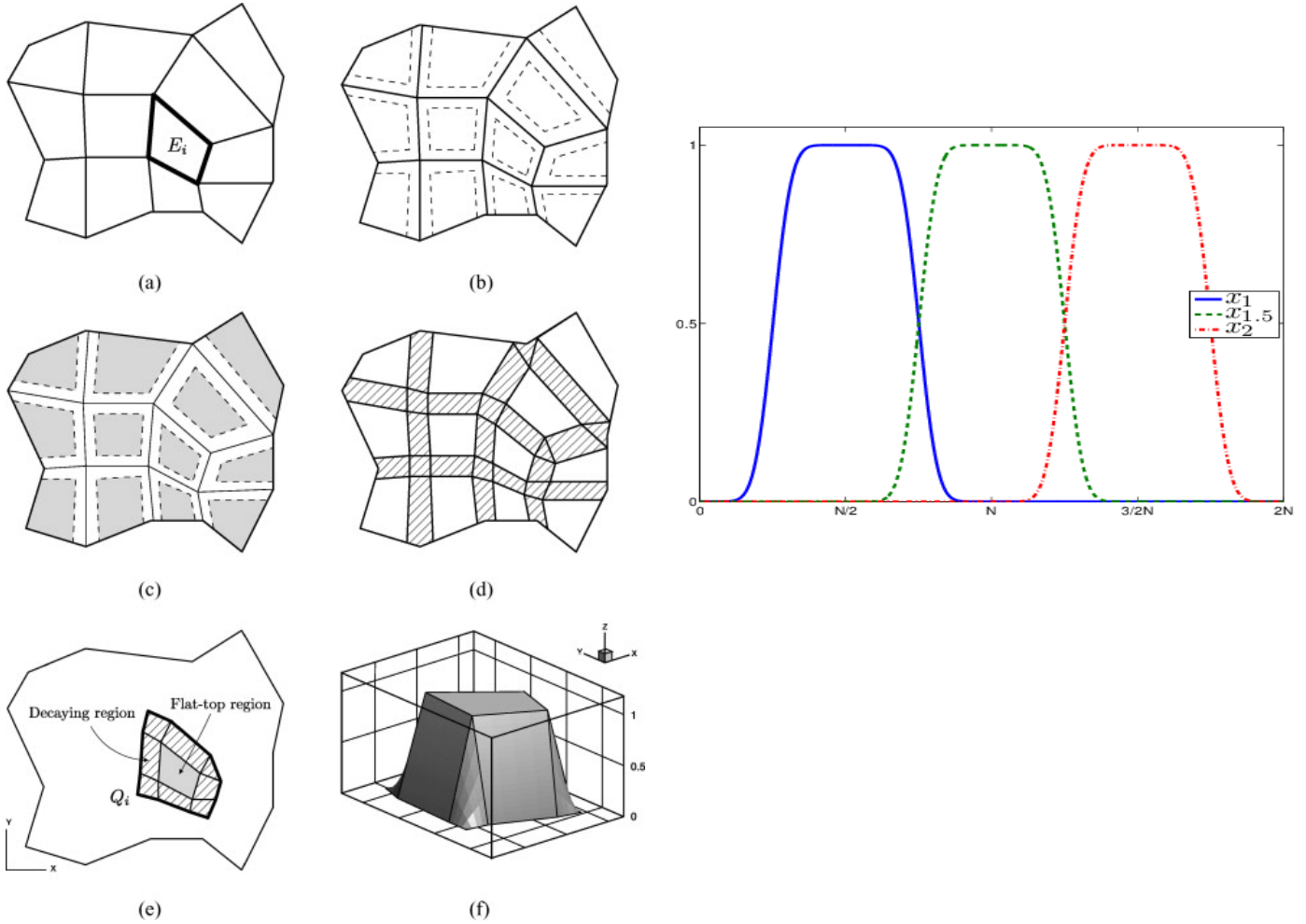
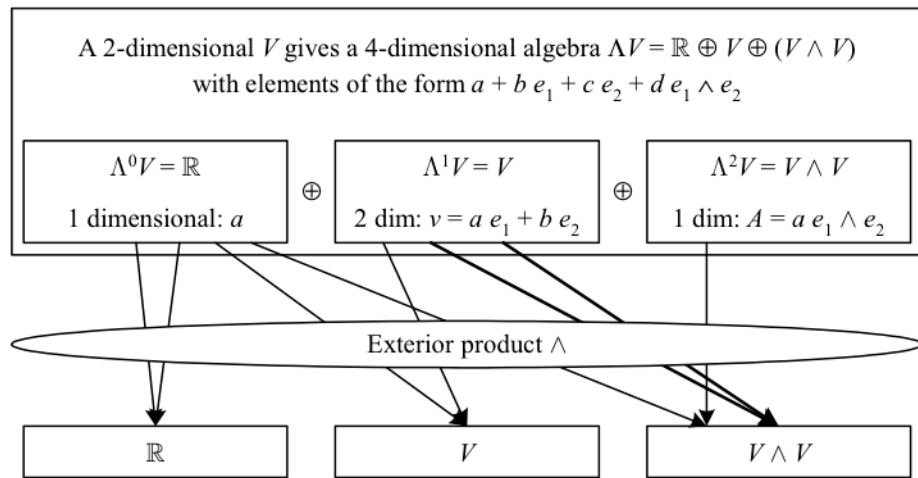


Partition of unity

A constructive approach



Example of an exterior algebra



$$v \wedge w = (a e_1 + b e_2) \wedge (c e_1 + d e_2) = (ad - bc) e_1 \wedge e_2$$

The exterior derivative

$$d(\omega + \eta) = d\omega + d\eta$$

$$d(\omega \wedge \eta) = d\omega \wedge \eta + (-1)^p \omega \wedge d\eta \quad (p = \text{deg } \omega)$$

$$d(d\omega) = 0.$$

Definition de Rham cohomology

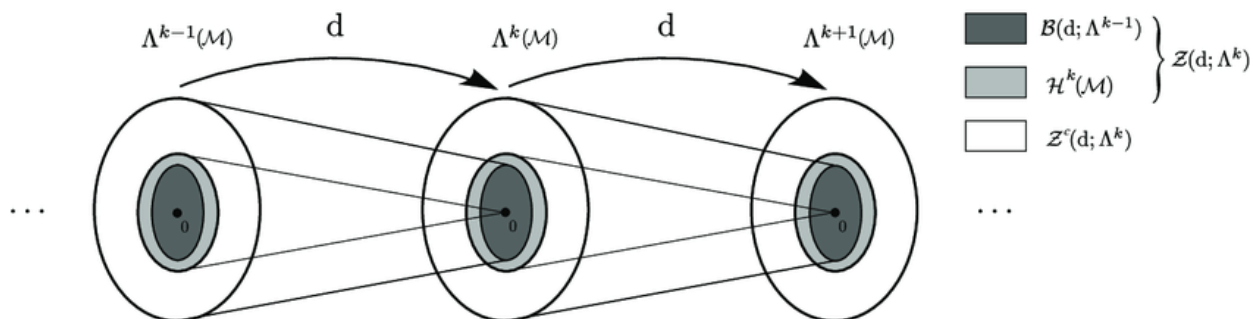
- Kernel and image of a map
- Space of all forms on a smooth manifold and exterior derivative

$$d : \Lambda^i(M) \rightarrow \Lambda^{i+1}(M) \quad d^2 = 0$$

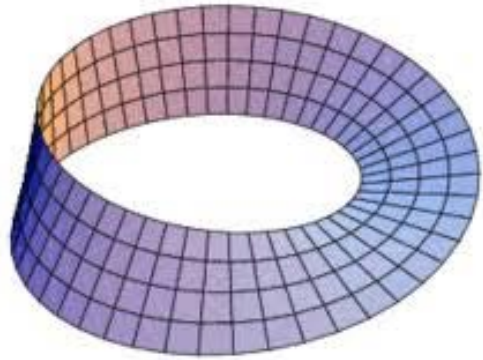
- Definition of de Rham cohomology:

$$H^i(M) := \frac{\ker(d : \Lambda^i \rightarrow \Lambda^{i+1})}{\text{im}(d : \Lambda^{i-1} \rightarrow \Lambda^i)}$$

- Can think of de Rham cohomology intuitively as counting the numbers of i-dimensional holes



Non orientable manifolds



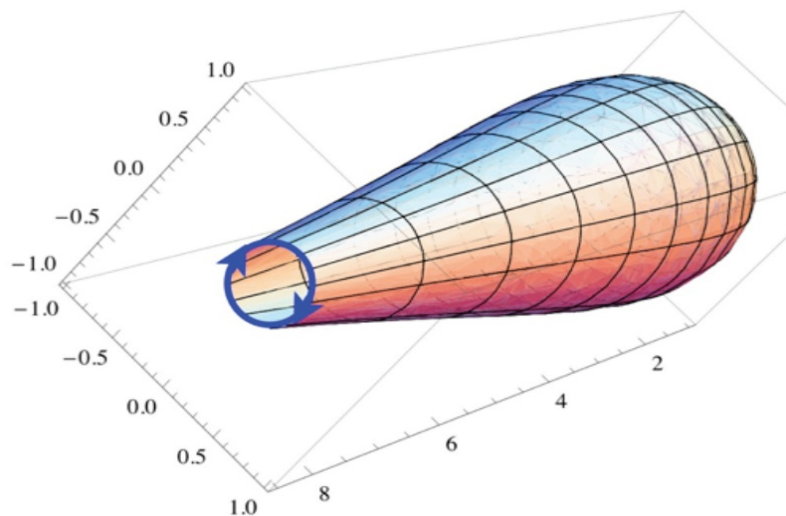
Manifold with boundary

Every point has a neighborhood that either:

- looks like a region in n -dimensional space, or
- looks like a region in n -dimensional half space.



A disk is a 2-manifold with boundary a circle.



Summary + keywords

- Extrinsic / intrinsic (differential geometry)
- Topological manifold open sets, neighborhood, homeomorphism, relative topology.
- Smooth manifold transition functions, atlas chart or local coordinate system, diffeomorphism.
- Tangent space $T_p(\mathcal{M})$, basis of the tangent space, coordinate frames, $\varphi_*^{-1}(\frac{\partial}{\partial x^j}|_{\varphi(p)})$.
- Vector fields One tangent vector at each point of \mathcal{M} , smooth vector fields $\mathfrak{X}(\mathcal{M})$, a vector field maps smooth function to smooth functions, local flow, integral curves, regular point.
- Tensor fields Tensor, symmetric, alternating $\Lambda(V)$, wedge product, dual space $\{(dx^j)_p\}_j$, a tensor at every point of \mathcal{M} , smooth tensor fields $\mathfrak{T}^k(\mathcal{M})$, $\Lambda(\mathcal{M})$ the exterior algebra.