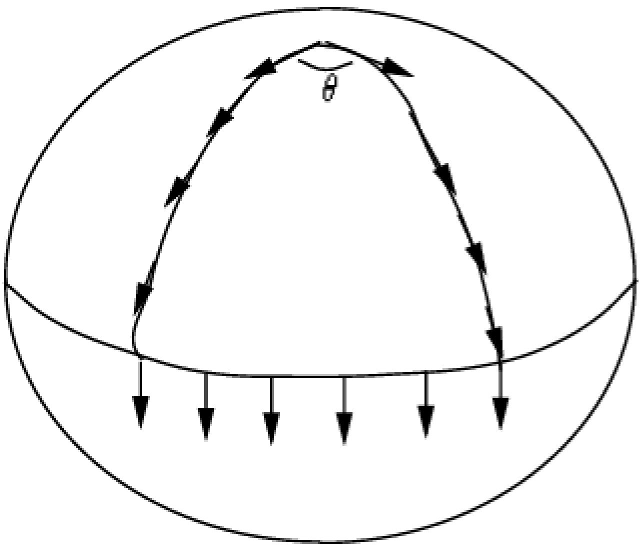
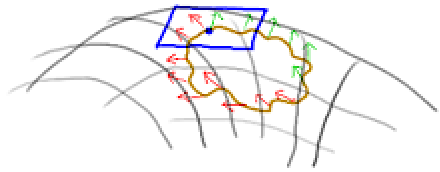
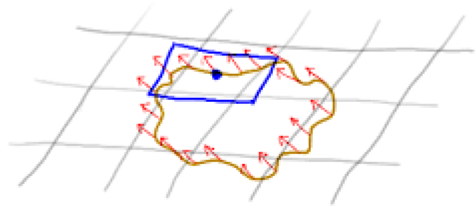
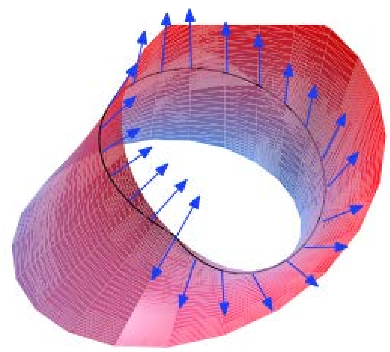
Parallel transport and holonomy map:

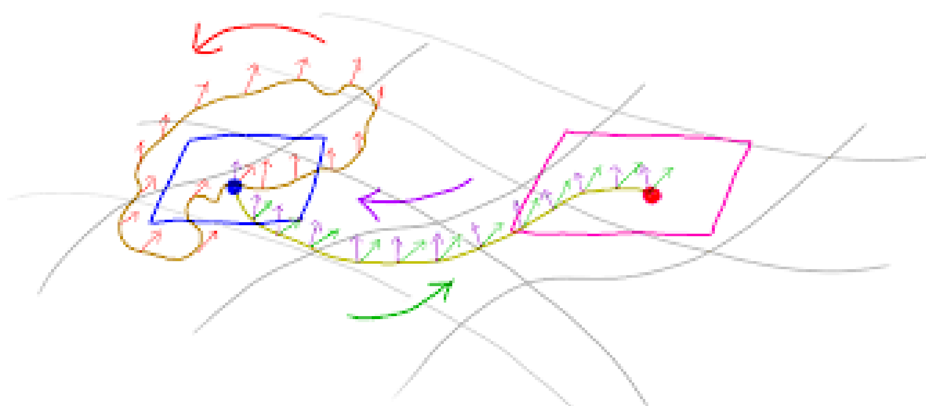








Independence with respect to initial point:



Only a few holonomy groups are possible:

Berger's classification

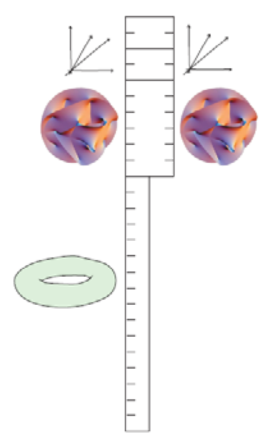
- Holonomy classification theorem by Berger (1955):

		Kähler	Calabi-Yau	Hyper-kähler	quat. Kähler	G_2-hol.	Spin(7)-hol.
	SO(n)	**U($\frac{n}{2}$)**	**SU($\frac{n}{2}$)**	**Sp($\frac{n}{4}$)**	**Sp(1)Sp($\frac{n}{4}$)**	**G_2**	**Spin(7)**
$n=2$:	✓	✓					
$n=3$:	✓						
$n=4$:	✓	✓	✓(2)				
$n=5$:	✓						
$n=6$:	✓	✓	✓(2)				
$n=7$:	✓					✓(1)	
$n=8$:	✓	✓	✓(2)	✓(3)	✓		✓(1)
$n=4m+1$:	✓						
$n=4m+2$:	✓	✓	✓(2)				for
$n=4m+3$:	✓						$m \geq 2$
$n=4m+4$:	✓	✓	✓(2)	✓$(m+2)$	✓		

- Ricci-flatness (red cases) is related to the existence of globally defined, covariantly constant spinors. ⇒ numbers in parenthesis

Special holonomy in compactification

- Classical usage: The equations for (partially) unbroken 4d SUSY after a Kaluza-Klein compactification reduce to the internal space admitting a parallel spinor, i.e. the internal space being Ricci-flat. (Candelas et al. 1985)
  ⇝ Calabi-Yau compactification
- More recently (following the same basic idea):
  - G_2-compactification of 10d string theory
    ⇒ 3d $N=1$ toy models
  - G_2-compactification of 11d M-theory
    ⇒ 4d $N=1$ models
  - Spin(7)-compactification of 11d M-theory
    ⇒ 3d $N=1$ toy models



- However, spaces with special holonomy can also be used in higher dimensional gauge theory.