Recall: $X = \mathbb{Z}^d$

Let $A := \{x-y \mid x, y \in X\}$, $a \in A$

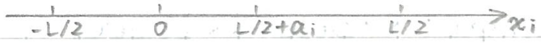
$N_L(a) := |\{x \in X \cap C_L \mid x - a \in X \cap C_L\}|$, $n_a := \lim_{L\to\infty} L^{-d} N_L(a)$

Set $x = \begin{pmatrix} x_1 \\ \vdots \\ x_d \end{pmatrix} \in {}^*\mathbb{Z}^d$, $a = \begin{pmatrix} a_1 \\ \vdots \\ a_d \end{pmatrix} \in \mathbb{Z}^d$, $N_L(a) = |\{x \in \mathbb{Z}^d \mid -\frac{L}{2} \le x_i \le \frac{L}{2}$ and $-\frac{L}{2} \le x_i - a_i \le \frac{L}{2}\}|$

$i = 1, 2, \cdots, d$

$(-\frac{L}{2} \le x_i \le \frac{L}{2}$ && $-\frac{L}{2} + a_i \le x_i \le \frac{L}{2} + a_i)$ (1)

If $a_i \ge 0$ then (1) $\Leftrightarrow -\frac{L}{2} + a_i \le x_i \le \frac{L}{2}$; If $a_i \le 0$ then (1) $\Leftrightarrow -\frac{L}{2} \le x_i \le \frac{L}{2} + a_i$

Only consider $a_i \le 0$ 

$(\frac{L}{2} \in \mathbb{R}, a_i \in \mathbb{Z})$

$\therefore N_{Li}(a) = 1 + 2[\frac{L}{2}] - |a_i|$, and this also works when $a_i > 0$ (symmetric)

N_{Li} is # of solutions on the i^{th} dimention

$\therefore N_L(a) = \prod_{i=1}^{d} (1 + 2[\frac{L}{2}] - |a_i|)$

$$\Rightarrow n_a = \lim_{L\to\infty} \prod_{i=1}^{d} (1 + 2[\tfrac{L}{2}] - |a_i|) L^{-d}$$

$$= \lim_{L\to\infty} \prod_{i=1}^{d} \left(\frac{1-|a_i|}{L} + \frac{2[\frac{L}{2}]}{L} \right) = \prod_{i=1}^{d} \lim_{L\to\infty} \left(\frac{1-|a_i|}{L} + \frac{2[\frac{L}{2}]}{L} \right)$$

$$= \prod_{i=1}^{d} \lim_{L\to\infty} \left(\frac{1-|a_i|}{L} + \frac{L+O(1)}{L} \right) = 1$$