
Homework 9

Exercise 1 *Consider the function* $\cos : (0, \pi) \to (-1, 1)$. *Show that this function is invertible and compute the derivative of its inverse. Same question for the function* $\tan : \big(-\frac{\pi}{2}, \frac{\pi}{2}\big) \to \mathbb{R}$, *and for the function* $\tanh : \mathbb{R} \to (-1, 1)$.

Exercise 2 *Determine the most general indefinite integral for the function* $\mathbb{R}^* \ni x \mapsto \frac{1}{x} \in \mathbb{R}$.

Exercise 3 *Find the area under the following curves between the given bounds:*

1. $x \mapsto x^3$ *between* $x = 1$ *and* $x = 2$,
2. $x \mapsto \mathrm{e}^{-x}$ *between* $x = 0$ *and* $x = b > 0$, *what happens when* $b \to \infty$ *?*
3. $x \mapsto \cos(x) + \cos(2x)$ *between* $x = 0$ *and* $x = \pi/4$,
4. $x \mapsto x - \sin(x)$ *between* $x = 0$ *and* $x = 1$,

and represent each of these areas on a drawing.

Exercise 4 *For any* $x \in \mathbb{R}$ *with* $x \neq -1$ *we consider the sequence* $(a_n)_{n\in\mathbb{N}}$ *given by*

$$a_n := \frac{x^n}{1 + x^n}.$$

For which x *does the limit* $\lim_{n\to\infty} a_n$ *exists ? Give the value of this limit whenever it exists.*

Exercise 5 (optional) *Let* $f : I \to \mathbb{R}$ *be a continuous function at* $b \in I$, *and let* g *be another function satisfying* $\lim_{x\to a} g(x) = b$. *Show that the following equality holds:*

$$\lim_{x\to a} f\big(g(x)\big) = f(b).$$