## Homework 3

**Exercise 1** *Let $f, g : \mathbb{R} \to \mathbb{R}$ be two continuous functions. Show as precisely as possible that*

1. *the sum $\lambda f + g$ is continuous on $\mathbb{R}$ for any $\lambda \in \mathbb{R}$,*
2. *the product $fg$ is continuous on $\mathbb{R}$,*

**Exercise 2** *Compute the following limits, it they exist:*

1. $\lim_{x \to 0_-} \left(\frac{1}{x} + \frac{1}{|x|}\right)$ *and* $\lim_{x \to 0_+} \left(\frac{1}{x} + \frac{1}{|x|}\right)$,
2. $\lim_{x \to 2_+} \frac{x^2+x-6}{|x-2|}$ *and* $\lim_{x \to 2_-} \frac{x^2+x-6}{|x-2|}$,
3. $\lim_{h \to 0} \frac{\sqrt{1+h}-1}{h}$.

**Exercise 3** *Let $I$ be an open interval in $\mathbb{R}$, and let $f : I \to \mathbb{R}$ be a continuous function. If $f(x) \neq 0$ for some $x \in I$, show that there exists $\delta > 0$ such that $f(x+h) \neq 0$ for any $h \in [-\delta, \delta]$.*

**Exercise 4** *Determine the slope of the tangent at each point of the graph of the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = 2x^2 - 3x + 2$.*

**Exercise 5** *For each positive integer $n$ consider the polynomial function $p_n : \mathbb{R} \to \mathbb{R}$ defined by $p_n(x) = x^n$ and show that*

$$p_n'(x) \equiv \frac{\mathrm{d}p_n}{\mathrm{d}x}(x) = nx^{n-1}.$$

*In your proof you can use the equality*

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

*with* $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.