Nagoya University, G30 program Fall 2016

Mathematics Tutorial Ia (calculus) Instructor : Serge Richard

---

Homework 7

Exercise 1 *Compute the following limits:*

(i) $\lim_{x\to 0} \frac{x-\sin(x)}{x^3}$,

(ii) $\lim_{x\to 0} \frac{x^2}{1+x-e^x}$.

Exercise 2 *a) Let* $f(x) = x^2\sin(1/x)$ *and* $g(x) = \sin(x)$ *for any* $x \in (-1,0)\cup(0,1)$. *Show that* $\lim_{x\to 0}\frac{f'(x)}{g'(x)}$ *does not exist, but that* $\lim_{x\to 0}\frac{f(x)}{g(x)} = 0$.

b) Explain how this example fits in with L'Hospital's rule ?

Exercise 3 *Show that* $\sin(x) \leq x$ *for any* $x \geq 0$.

Exercise 4 *Consider the functions hyperbolic cosine* $\cosh : \mathbb{R}\to\mathbb{R}$ *and hyperbolic sine* $\sinh : \mathbb{R}\to\mathbb{R}$ *defined by the formulas*

$$\cosh(x) := \frac{e^x + e^{-x}}{2} \qquad \textit{and} \qquad \sinh(x) := \frac{e^x - e^{-x}}{2}.$$

Compute the derivative of these functions and sketch the graph of the functions cosh *and* sinh. *Prove the following relation:*

$$\cosh(x)^2 - \sinh(x)^2 = 1, \qquad \forall x \in \mathbb{R}.$$

Exercise 5 *Find the critical points for the differentiable functions* $f : \mathbb{R}\to\mathbb{R}$ *defined for* $x\in\mathbb{R}$ *by*

$$a)\ -x^2+2x+2, \qquad b)\ x^3-3, \qquad c)\ \cos(x), \qquad d)\ \sin(x)+\cos(x).$$

Exercise 6 *Find the point of the curve of equation* $y^2 = 4x$ *which is the nearest one to the point* $(2,3)$.

Exercise 7 *Determine the maximal domain on which the following functions are defined and sketch their graph as precisely as possible:*

$$a)\ f(x) = \frac{x-3}{x^2+1}, \qquad b)\ g(x) = \frac{2x^2-1}{x^2-2}, \qquad c)\ h(x) = x + \frac{1}{x}\ .$$