
Homework 6

Exercise 1 By using that $\sin'(x) = \cos(x)$ show that $\cos'(x) = -\sin(x)$ for any $x \in \mathbb{R}$.

Exercise 2 1) Compute the derivative of the function $(0, \infty) \ni x \mapsto \frac{1}{x} \in (0, \infty)$.

2) For any $q \in \mathbb{Q}$ with $q > 0$ let us define the function P_{-q} by $P_{-q}(x) \equiv x^{-q} := \frac{1}{x^q}$. With the previous statement together with the content of Exercise 3 of Homework 5, show that

$$P'_{-q}(x) = -qx^{-q-1} \equiv -q\frac{1}{x^{q+1}}.$$

Exercise 3 Compute the derivative of the functions $f : \mathbb{R} \rightarrow \mathbb{R}$ defined for $x \in \mathbb{R}$ by

$$a) \sin((2x^2 - 3)^2) \quad b) \frac{(x+3)^3}{(2x-3)^2 + 1} \quad c) \frac{1}{\sin(3x)^2 + 1}$$

Exercise 4 Compute the derivatives of order 1, 2 and 3 for the functions $f : \mathbb{R} \rightarrow \mathbb{R}$ defined for $x \in \mathbb{R}$ by

$$a) \cos(x) \quad b) \cos(x)\sin(x) \quad c) x^4 + x^3 + x^2 + x^1 + 1$$

Exercise 5 Consider the curve in \mathbb{R}^2 defined by the relation

$$F(x, y) = x^3 + y^3 - 6xy = 0.$$

1. Find the equation of the tangent line at the point $(3, 3)$,
2. For $x, y > 0$, at what point (x, y) is the tangent to the curve a horizontal line ?