
Homework 3

Exercise 1 Let $f, g : (0, \infty) \rightarrow \mathbb{R}$ having a limit $f_0 \in \mathbb{R}$ and $g_0 \in \mathbb{R}$ at $x = 0$, i.e. $\lim_{x \rightarrow 0} f(x) = f_0$ and $\lim_{x \rightarrow 0} g(x) = g_0$. Show as precisely as possible that

1. $\lim_{x \rightarrow 0} (f(x) + g(x)) = f_0 + g_0$,
2. $\lim_{x \rightarrow 0} f(x)g(x) = f_0 g_0$.

Exercise 2 Compute the following limits, if they exist:

1. $\lim_{x \rightarrow 0^-} \left(\frac{1}{x} + \frac{1}{|x|} \right)$ and $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} + \frac{1}{|x|} \right)$,
2. $\lim_{x \rightarrow 2^+} \frac{x^2+x-6}{|x-2|}$ and $\lim_{x \rightarrow 2^-} \frac{x^2+x-6}{|x-2|}$,
3. $\lim_{h \rightarrow 0} \frac{\sqrt{1+h}-1}{h}$.

Exercise 3 Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be two continuous functions. Show as precisely as possible that

1. the sum $\lambda f + g$ is continuous on \mathbb{R} for any $\lambda \in \mathbb{R}$,
2. the product fg is continuous on \mathbb{R} ,

Exercise 4 Let I be an open interval in \mathbb{R} , and let $f : I \rightarrow \mathbb{R}$ be a continuous function. If $f(x) \neq 0$ for some $x \in I$, show that there exists $\delta > 0$ such that $f(x+h) \neq 0$ for any $h \in [-\delta, \delta]$.

Exercise 5 Determine the slope of the tangent at each point of the graph of the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 2x^2 - 3x + 2$.