
Homework 13

Exercise 1 a) Provide the Taylor's expansion (for x around 0) of order n for the following functions, and provide an estimate on the remainder term:

1) $(-1, \infty) \ni x \mapsto \ln(1+x) \in \mathbb{R}$,

2) $(-1, 1) \ni x \mapsto (1+x)^s \in \mathbb{R}$ for any $s \in \mathbb{R}$,

3) $\mathbb{R} \ni x \mapsto \sin(x) \in \mathbb{R}$,

4) $\mathbb{R} \ni x \mapsto \sinh(x) \in \mathbb{R}$,

5) $\mathbb{R} \ni x \mapsto \cosh(x) \in \mathbb{R}$,

b) Provide the Taylor's expansion (for x around 0) of order 3 for the function

$$(-1, 1) \ni x \mapsto \ln\left(\frac{1+x}{1-x}\right) \in \mathbb{R}.$$

What about the terms with an even power of x when we write its Taylor's expansion of order n for arbitrary n ?

Exercise 2 Provide the Taylor's expansion (for x around $\pi/4$) of order n for the functions mentioned below, and provide an estimate on the remainder term:

$$\mathbb{R} \ni x \mapsto \cos(x) \in \mathbb{R}.$$

Exercise 3 By using Taylor's expansions compute the following limits:

1. $\lim_{x \rightarrow 0} \frac{\sin(x^2)}{x \tan(x)}$,

2. $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{\sin(x)}$,

3. $\lim_{x \rightarrow 0} \frac{\sin(x) - e^x + 1}{x}$.