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**Homework 10**

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**Exercise 1** Find the area under the following curves between the given bounds:

1.  $x \mapsto x^3$  between  $x = 1$  and  $x = 2$ ,
2.  $x \mapsto e^{-x}$  between  $x = 0$  and  $x = b > 0$ , what happens when  $b \rightarrow \infty$  ?
3.  $x \mapsto \cos(x) + \cos(2x)$  between  $x = 0$  and  $x = \pi/4$ ,
4.  $x \mapsto x - \sin(x)$  between  $x = 0$  and  $x = 1$ ,

and represent each of these areas on a drawing.

**Exercise 2** Write out the lower and the upper Riemann sums for the function  $x \mapsto x^2$  in the interval  $[0, 2]$ . Use a regular partition of the interval divided into  $n$  subintervals of the same length. The following formula can be used:

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

What happens when  $n \rightarrow \infty$  ?

**Exercise 3** Consider the function  $[0, 1] \ni x \mapsto e^x \in \mathbb{R}$ , and consider a regular partition of  $[0, 1]$  divided into  $n$  intervals of length  $\frac{1}{n}$ . Compute the following Riemann sums:

1.  $I_l := \sum_{j=0}^{n-1} \frac{1}{n} e^{\frac{j}{n}}$  left rule,
2.  $I_r := \sum_{j=1}^n \frac{1}{n} e^{\frac{j}{n}}$  right rule,
3.  $I_m := \sum_{j=0}^{n-1} \frac{1}{n} e^{\frac{j+1/2}{n}}$  midpoint rule,
4.  $I_{tri} := \frac{1}{2}(I_l + I_r)$  trapezoidal rule,
5.  $I_{Sim} := \sum_{j=0}^{n/2-1} \frac{1}{3n} (e^{\frac{2j}{n}} + 4e^{\frac{2j+1}{n}} + e^{\frac{2j+2}{n}})$  for  $n$  even Simpson's rule.

Can you illustrate these rules on a drawing and compare their rate of convergence to their limit as  $n \rightarrow \infty$  ? The following formula can be used for any  $a > 0$  with  $a \neq 1$ :

$$\sum_{k=0}^{m-1} a^k = \frac{a^m - 1}{a - 1}.$$