Nagoya University, G30 program Fall 2015

Mathematics Tutorial Ia (calculus) Instructor : Serge Richard

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Homework 9

Exercise 1 *Find the area under the following curves between the given bounds:*

1. $x \mapsto x^3$ *between* $x = 1$ *and* $x = 2$,
2. $x \mapsto \mathrm{e}^{-x}$ *between* $x = 0$ *and* $x = b > 0$, *what happens when* $b \to \infty$?
3. $x \mapsto \cos(x) + \cos(2x)$ *between* $x = 0$ *and* $x = \pi/4$,
4. $x \mapsto x - \sin(x)$ *between* $x = 0$ *and* $x = 1$,

and represent each of these areas on a drawing.

Exercise 2 *Write out the lower and the upper Riemann sums for the following functions and intervals. Use a regular partition of the interval divided into* n *subintervals of the same length.*

1. $x \mapsto x^2$ *in the interval* $[0, 2]$,
2. $x \mapsto 1/x$ *in the interval* $[1, 2]$.

Exercise 3 *Use the Riemann sums for a suitable function to show that for any* $n \in \mathbb{N}^*$

$$\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n} \leq \ln(2) \leq \frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n-1}.$$

Exercise 4 *By using Riemann sums, compute the area under the curve* $x \mapsto x^2$ *between* $x = 0$ *and* $x = 1$. *The following formula can be used:*

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Exercise 5 *Consider the function* $[0, 1] \ni x \mapsto \mathrm{e}^x \in \mathbb{R}$, *and consider a regular partition of* $[0, 1]$ *divided into* n *intervals of length* $\frac{1}{n}$. *Compute the following Riemann sums:*

1. $I_l := \sum_{j=0}^{n-1} \frac{1}{n} \mathrm{e}^{\frac{j}{n}}$ left rule,
2. $I_r := \sum_{j=1}^{n} \frac{1}{n} \mathrm{e}^{\frac{j}{n}}$ right rule,
3. $I_m := \sum_{j=0}^{n-1} \frac{1}{n} \mathrm{e}^{\frac{j+1/2}{n}}$ midpoint rule,
4. $I_{tri} := \frac{1}{2}\big(I_l + I_r\big)$ trapezoidal rule,
5. $I_{Sim} := \sum_{j=0}^{n/2-1} \frac{1}{3n} \big(\mathrm{e}^{\frac{2j}{n}} + 4\mathrm{e}^{\frac{2j+1}{n}} + \mathrm{e}^{\frac{2j+2}{n}}\big)$ *for* n *even* Simpson's rule.

Can you illustrate these rules on a drawing and compare their rate of convergence to their limit as $n \to \infty$? *The following formula can be used for any* $a > 0$ *with* $a \neq 1$:

$$\sum_{k=0}^{m-1} a^k = \frac{a^m - 1}{a - 1}.$$